Observational evidence for earthquakes as a nonlinear dynamic process

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Abstract

Herein I review experimental evidence for earthquake scale-invariance. Earthquake occurrence exhibits scaling properties: the temporal correlations of earthquakes are power law and the distribution of the earthquake size is also a power law. Recently, it has been determined that several other statistical features of earthquakes, i.e., spatial distribution of earthquakes, rotation of their focal mechanisms, and stress patterns which both cause and are caused by earthquakes, are also scale-invariant. Seismicity is also recognized as a chaotic phenomenon. The intrinsic randomness of an earthquake occurrence makes most of standard physical techniques unsuitable for study, thus the methods of stochastic processes should be used for the seismicity analysis. I present evidence that seismicity is controlled by scale-invariant statistical distributions which possibly have universal values for exponents. Finally, I discuss mechanical and other models proposed to reproduce these properties of seismicity, and offer a model of random defect interaction which, without additional assumptions, seems to explain most of the available empirical results.

1. Introduction

Since the late 19th century it has been known that an earthquake occurrence exhibits scale-invariant properties: Omori (1895) was the first investigator to note that the aftershock rate decays as a power law with time. (In Section 2 are the definitions of some seismological terms.) Ishimoto and Iida (1939) and later Gutenberg and Richter (1944) found that earthquake size distribution also follows a power law. Recently many other earthquake empirical distributions have been found to be scale-invariant or fractal: Takayasu (1990, p. 31), Kagan (1992a), and Kagan and Jackson (1991a) list several attempts to determine scale-invariant features of earthquakes. At this stage of our seismicity investigations I believe that it is no longer acceptable to reveal the scale-invariance of certain earthquake distributions and determine their fractal exponents (or dimensions): such determination should be accompanied by an analysis of possible errors, as well as by establishing a possible universality of observed fractal features. One should carefully read the criticism by Kadanoff (1986) and by Ruelle (1991, pp. 71-73): due to the lack of organizing theory it is difficult to judge the intrinsic merits of each empirical measurement.

The study of an earthquake occurrence...
presents several serious difficulties: (1) there is no comprehensive theory of earthquake rupture; even basic relations cannot logically be derived from fundamental physical laws; (2) earthquakes occur in the deep interior of the Earth, i.e., in places which are largely inaccessible to direct observations and measurements; (3) experiments with earthquakes are not possible; (4) the modelling of fracture either in a laboratory or by computer simulation is still in a very rudimentary stage; it is not clear whether the results of such modelling are applicable to natural earthquakes; (5) earthquake occurrence is characterized by extreme randomness, the stochastic nature of seismicity is not reducible by more numerous or more accurate measurements; (6) the degree of seismicity randomness is much greater than that of atmospheric turbulence: whereas it is possible to predict weather with increasing accuracy, as the time lag between an observation and the forecast decreases, an earthquake usually strikes without any warning.

The properties of seismicity described above, allow us to compare it with the turbulence of a fluid flow (see Kagan, 1992a): (a) they both share inherent randomness; (b) their major statistical ingredients are scale-invariant; (c) they both have hierarchically organized structures; (d) the size of major structures, which control deformation patterns, in both cases is comparable to the maximum size of the region, these coherent systems represent a unique object for a scientific study; (e) the scale of self-similar structures extends over many orders of magnitude, i.e., for turbulence from submillimeter scales to intergalactic distances, for seismicity, again from about a millimeter to thousands of kilometers; (f) both phenomena are intermittent in time and space, although the degree of seismic intermittency as reflected in its fractal exponents, is higher than that of fluid turbulence; (g) both are time-irreversible; and, finally, (h) both are complex continuum-mechanical systems with very large (potentially infinite) numbers of degrees of freedom. Of course, there are many differences between these phenomena as well: (a) the medium of deformation is fluid in the first case and solid in the latter case; complex interaction of gaseous, liquid and solid phases in earthquake focal zones is believed to significantly influence the earthquake occurrence; (b) the linear elastic deformation of an isotropic solid body is characterized by two constants versus one constant for fluids, hence continuum-mechanical behavior of solids is significantly more complex than that of fluids; (c) most earthquake deformation is the effect of dislocations (translational defects), whereas disclinations (rotational defects) play a subordinate role (Edelen and Lagoudas, 1988); in turbulent motion of fluid, vortices (disclinations) are primary vehicles of deformation. However, we believe that eventually the comparison of the two modes of condensed matter deformation will yield significant new insight into the mechanics of both phenomena.

The intrinsic randomness of seismicity means that most standard techniques would not produce the expected results when applied to earthquake data. For example, case studies of particular earthquakes or earthquake sequences present an insufficient evidence, since if the phenomenon is stochastic, in a certain sense, any earthquake sequence is possible. Thus, unless it can be shown that such cases are proper representatives of the general earthquake population, the evidence based on case studies alone cannot be taken as final word: by using a biased selection rule in a largely random earthquake population, one can obtain almost any kind of results. Hence an earthquake occurrence has to be explored by statistical methods. Even statistical analysis of seismicity presents a rather difficult problem: in testing for a possible variation of parameters we cannot use standard statistical techniques which presuppose that the variables in question are either independent or if they are dependent, the correlation radius is much smaller (in time or in space) than the span of a catalog. The seismicity is known, how-
ever, to display very long-term and long-range correlations, thus the only solution feasible in this situation is to check for the consistency of the results: if the same statistical feature is observed in various catalogs obtained with different observation techniques, in various seismic regions, or time limits, we can be confident that the correlation has a physical basis.

Similar problems arise when one attempts to study variations of seismicity parameters (exponents) over different tectonic regions, various temporal, spatial, and earthquake size scales. In most such studies the initial result is that the parameters do vary, thus there is no universal value for the parameter in question. This presents a major challenge in the search for the physical theory of seismicity: either the earthquake occurrence is governed by some universal laws which call for a physical model or a theory, or no such laws are available, at least at this stage of our investigation, and we are limited to a phenomenological description. In other words, we need to find out, in any empirical result, whether it is a property of earthquake occurrence in general, or a description of a particular seismogenic region.

In many such cases it is possible to show that the parameter variation is a statistical artifact caused either by a substandard processing technique, by biased selection, or some other factors. However, in some cases the variation seems real. Thus we are confronted with a dilemma: is it a physical property of seismicity or is it due to some very long-term, but still random, fluctuations of the process. In this respect the analogy with fluid turbulence is again useful. The velocity of rock material motion in plate tectonics is about $10^{10}$ times smaller than similar velocities of the Earth's atmosphere. If we, for example, measure air velocity in a cloud of one km size, the results would probably be influenced by the size of the cloud, but one should not claim that this is the property of a general turbulent flow of the atmosphere, since the cloud will disappear or significantly change in a few minutes. However, for seismic deformation of rock material the time scales involved are much longer, of the order of millions of years, hence one could easily confuse the random fluctuations of a stochastic process with the time-invariant properties of the seismicity. Observations of seismicity extend only a few decades for the instrumental catalogs (since the beginning of 20th century), and several hundred years for historical and paleo-seismic data. Thus only a very limited time 'slice' of earthquake occurrence is available for quantitative study. In this work we assume that the seismic process is ergodic, therefore, given the shortness of the seismic record, we test the seismicity models using an ensemble of seismic zones, rather than an ensemble of occurrence times within a single zone.

2. Earthquakes

Since this paper is intended both for seismologists and non-seismologists, in this section I briefly describe earthquakes and earthquake catalogs as primary objects of statistical study. As a first approximation, an earthquake may be represented by a sudden shear planar failure – appearance of a large dislocation loop (Aki and Richards, 1980) in rock material. Fig. 1a shows a fault-plane trace on the surface of the Earth (similar to viewing from above an earthquake occurring on the San Andreas fault). Earthquake rupture starts at the point on the fault-plane called the hypocenter (the epicenter is a projection of the hypocenter on the Earth's surface), and propagates with velocity close to that of shear waves (3.0–4.5 km/s). The centroid is in the center of the ruptured area, its position being determined by seismic moment tensor inversion (Dziewonski et al., 1993, and references therein). As a result of the rupture two sides of the fault surface are displaced relative to each other in the direction of the arrows; for large earthquakes the displacement is of the order of a few meters.

Modern plate tectonics, formulated in the 1960s, explain the earthquake occurrence as a
The earthquake rupture excites seismic waves which are registered by seismographic stations. In the absence of internal boundaries in the homogeneous and isotropic rock material, two types of waves propagate from the source: primary, compressional (P-waves) and secondary, shear (S-waves). The best modern stations provide digital registration of three components of the ground motion in a very broad frequency range. The seismograms obtained in this way are processed by special computer programs to obtain properties of earthquakes. Preliminary seismogram inversion can be accomplished in about an hour after the earthquake occurrence.

The routine results of seismogram inversions characterize earthquakes by origin time, hypocenter (centroid) position, and by the second-rank symmetric seismic moment tensor for each event. In the system of coordinates shown in Fig. 1b the tensor matrix is

\[
M = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 
\end{bmatrix},
\]

where \( M \) is a scalar seismic moment of an earthquake, measured in Newton-m (Nm). In an arbitrary system of coordinates all entries in (1) are non-zero. However, the tensor is always traceless, with a zero determinant: hence it has only four degrees of freedom – one for the norm of the tensor (scalar seismic moment) and three for orientation (they define the focal mechanism of an earthquake). Another equivalent representation of the earthquake focus is a quadrupole source of a particular type, known in seismology as a double-couple (Aki and Richards, 1980).

Figs. 1b,c show two graphic representations of an earthquake source: double-couple configuration of equivalent forces, and quadrupolar radiation patterns characteristic of earthquakes. The focal plots which are standard in seismology, involve painting on a sphere the sense of the first motion of P-waves, solid for compressional motion and open for dilatational. Two orthogonal
planes separating these areas are the fault and auxiliary planes. In routine determination of focal mechanisms it is impossible to distinguish between these planes. The intersection of these planes is the null-axis (N-axis), the P-axis is in the middle of the open lune, and the T-axis in the middle of the closed lune. These three axes are called principal axes of an earthquake focal mechanism, and their orientation defines the mechanism.

Magnitude \((m)\) is an empirical measure of the earthquake size (approximately proportional to the logarithm of the scalar seismic moment)

\[
m \simeq \frac{2}{3} \log_{10} M - 6. \tag{2}
\]

In the formula above, moment is measured in Nm. The equation calculates the moment magnitude, \(m_w\) (Kanamori, 1977); this magnitude will be used throughout the paper, unless specified otherwise. Another measure of earthquake size, the released seismic energy, is not evaluated directly and is a scalar, thus the seismic moment tensor is a preferred quantity for earthquake characterization. Data prior to the introduction of worldwide earthquake digital recording (i.e., before 1977), appeared in a more restricted form: earthquake focal mechanism had not been determined, at least not routinely, and magnitude was used to characterize earthquake size.

The scalar seismic moment corresponds to the normalized amplitude of seismic waves at the frequency close to zero; accuracy of its determination is higher than that of magnitudes, which are normalized amplitudes of waves at periods like 20 s, 1 s, etc. Random heterogeneities of the Earth’s structure at the wavelengths corresponding to these periods, significantly reduce the accuracy of magnitude determination: whereas the moment magnitude has a standard error of about 0.2 (or the moment varies by a factor of two), magnitudes have the accuracy of 0.3–0.5.

Earthquakes are usually subdivided by the depth of their focus into shallow (depth 0–70 km), intermediate (71–300 km), and deep (301–700 km) events. About 90% of shallow earthquakes and almost all deeper earthquakes are concentrated at subduction zones (Pacheco and Sykes, 1992). Shallow events exhibit strong clustering in time, large earthquakes are usually followed by sequences of smaller earthquakes called aftershocks; strong aftershocks, in their turn, often have their own sequences of events, etc. Occasionally, large earthquakes are preceded by foreshock sequences consisting of one or several weaker events. The number of foreshocks is much smaller than the number of aftershocks, thus the seismic process is highly asymmetric in time. The large event which dominates the sequence is called the mainshock.

The earthquake data are assembled in catalogs of which the most extensive, complete, and homogeneous at present is the global catalog of moment tensor inversions compiled by the Harvard group (Dziewonski et al., 1993). This catalog used in most of the calculations in this paper covers the period from January 1, 1977, to December 31, 1992, and contains more than 10,000 events. There are other catalogs of earthquake focal mechanisms, for example Sipkin and Needham (1992) compiled the global catalog which contains about 1300 events. Some of computations reported below are carried out using the Sipkin and Needham (1992) dataset; the results are consistent with those obtained using the Harvard list. In principle, if we know these earthquake parameters, we can calculate the low frequency motion they excite, and henceforth the deformation history of the Earth’s brittle crust. Therefore in modern catalogs we have a complete representation of the seismic process, at least in the low frequency limit.

Even a casual inspection of earthquake catalogs reveals the random nature of earthquake occurrence, thus making it imperative to use statistical methods for their interpretation. As an example of earthquake data, Fig. 2 displays focal mechanisms for the earthquakes in Southern California. As the initial dataset we use Ellsworth (1990) historical/instrumental earth-
Earthquake focal mechanisms in S. California 1850-1994

Fig. 2. Focal mechanisms of earthquakes from the 1850–1994 modified Ellsworth list in the Southern California area and major surface faults. Lower hemisphere diagrams of focal spheres are shown, the diagrams can be thought of as 3-D rotations of the mechanism shown in Fig. 1c. Symbol size is proportional to earthquake magnitude. Solid 'beachballs' correspond to point sources, striped symbols to extended sources.

Quake catalog. We have modified it by adding: (1) the recent earthquakes from the Harvard catalog, including the preliminary solutions for the Northridge, California, 1994 earthquake and its strongest aftershock, (2) the focal mechanism solutions and spatially distributed seismic moment from other available publications, and (3) for earthquakes in the 19th century, we used the fault trace information and slip distribution for the largest earthquakes, to infer their distributed moment tensor. (Additional catalog details will be provided in D.D. Jackson and Y.Y. Kagan, 1994, manuscript in preparation).

It is obvious from Fig. 2, that earthquakes are not concentrated on a few faults and the mechanisms of neighboring events may have very different orientations, i.e., they undergo large 3-D rotations.

There are many other methods of earthquake study: detailed analysis of seismograms reveals, for example, a complex internal temporal and spatial structure of earthquake sources. Geological investigations of earthquake fault traces similarly disclose their complicated geometry, and geodetic observations allow us to study pre- and postseismic deformation, caused by earthquakes, etc. Unfortunately, these data usually lack uniform coverage of all events which makes them less suitable for statistical analysis.

Fig. 3 shows the cumulative stress change of the first stress invariant, $I_1$, (average normal stress – Jaeger and Cook, 1979) for Southern California from 1850 to the present (February 1, 1994). The distribution of the stress is dominated by several large earthquakes: 1857 Ft. Tejon, 1872 Owens Valley, 1952 Kern County, and 1992 Landers events. The incremental stress pattern forms a complex mosaic due to the interaction of incremental stress fields of many earthquakes (cf. Stein et al., 1992). The compli-
Incremental stress (invariant $I_1$) in S. California 1850-1994

Fig. 3. Incremental stress in Southern California. The modified Ellsworth catalog 1850–1994 is used. Stress is evaluated at a surface. Stress invariant $I_1$ (average normal stress) is shown.

cated character of stress once again underscores the necessity of analyzing the data statistically.

In our work of the last 15 years we have statistically analyzed several catalogs of tectonic earthquakes in order to study the interrelations between earthquakes and to further theoretical understanding of the earthquake process. The information available in earthquake catalogs suggests the general type of models which should be used to describe seismicity – stochastic, multidimensional, tensor-valued, point process: $M \times \mathbb{R}^3 \times T \times SO(3)$, where $\mathbb{R}^3$ is the 3-D Euclidean space, $T$ is time, and $SO(3)$ is the group of 3-D rotations. The results of this statistical analysis are reported below.
3. Size distribution – \( M \)

The distribution of earthquake sizes is usually invoked as a first confirmation for virtually any model of seismicity. Moreover, this distribution is by far the most studied feature of statistical seismology. Starting with its first discussion by Ishimoto and Iida (1939) and by Gutenberg and Richter (1944), it has been established that the number of earthquakes increases as a power law as their sizes decrease. This relation is usually referred to as the Gutenberg–Richter (G–R) law, and the power-law exponent is commonly known as the ‘\( b \)-value’. A very large body of literature exists concerning the size distribution, its interpretation and possible correlation with geotectonics, stress, rock properties, etc. For example, a search of the INSPEC database for the keywords like ‘earthquake size distribution’, ‘earthquake \( b \)-value’, yields about 200 publications in the last 4.5 years. However, I believe that proliferation of these investigations has not led to a deeper understanding of the earthquake generation process.

Almost all of the above-mentioned publications have studied earthquake magnitude distribution. There are many types of magnitudes in existence, each with its own \( b \)-value, and the \( b \)-values usually show a great variation (Kagan, 1991b). However, when one uses the seismic moment to analyze the size distribution, the random fluctuations of \( b \)-values are significantly reduced: they become statistically insignificant. Unfortunately, at present, the seismic moment tensor can be routinely determined only for relatively large earthquakes (Dziewonski et al., 1993; Sipkin and Needham, 1992). Since each magnitude can be evaluated only in a limited range of seismic wave amplitudes, in order to extend the dynamic range and compare the results obtained from various frequency ranges, complicated regression formulas to transform one magnitude into another have been proposed and used. As a result, significant systematic and random errors accumulate in the statistical distributions of earthquake magnitude. These errors together with suboptimal statistical estimation procedures, often biased selection, and general lack of rigor contribute to the results being of uncertain quality. Although almost all such measurements display a general power-law relation, many of the magnitude-frequency plots show deviations from the strict scale-invariance, these deviations often being interpreted as a proof of a certain preferred scale in earthquake generation, or a total breakdown of the self-similarity. Below I briefly review some of the claims of the scale-invariance breakdown.

The G–R relation can be transformed into a power-law (Pareto) distribution for the scalar seismic moment with the density \( \phi(M) \propto M^{-1-\beta} \), with \( \beta \approx \frac{3}{2}b \) (see Eq. 2). (I note that in many publications, especially in Physics journals, \( b \) and \( \beta \) are confused.) Statistical analysis of magnitude and seismic moment distributions yield the value of \( \beta \) between 1/2 and 1.0 for small and medium-size earthquakes. Simple considerations of finiteness of the seismic moment or deformational energy, available for an earthquake generation, require that the power-law relation be modified at the maximum size end of the moment scale (Kagan, 1991b). At the minimum, the distribution density tail must have a decay stronger than \( M^{-1-\beta} \) with \( \beta > 1 \). Usually this problem is solved by the introduction of an additional parameter, a ‘maximum moment’ \( (M_{\text{max}}) \), to the distribution. This new parameterization of the modified G–R relation has several forms (Kagan, 1991b; 1993). All these distributions presume that the major part (about 50% or more) of the total seismic moment is released by very large earthquakes, i.e., events with \( (M_{\text{max}}/10) < M \).

We assume that the seismic moment is distributed according to the gamma distribution (Kagan, 1991b) with a probability density:

\[
\phi(M) = C^{-1}M^{-1-\beta}\exp(-M/M_{\text{max}}),
\]

where \( C \) is a normalizing coefficient. To illustrate the gamma distribution fit to actual data,
Fig. 4a displays a cumulative histogram for the scalar seismic moment of the events in the Harvard catalog (Dziewonski et al., 1993), for shallow, intermediate, and deep earthquakes. For this plot the end of the catalog is August 31, 1993. To ensure the data uniformity in time and space we use events with $M \geq 10^{17.7}$ Nm, which correspond to $m \geq 5.8$. All of the curves display a scale-invariant segment (linear in the log-log plot) for small and intermediate values of the seismic moment. At large $M$, the curves are bent downwards, the lack of very strong earthquakes is the result of the above-mentioned finiteness of the seismic moment flux.

The maximum likelihood procedure (Kagan, 1991b) allows us to retrieve the values of parameters for all three earthquake categories: shallow events have $\beta = 0.62-0.70$; $M_{\text{max}} = 1.0 \times 10^{21}-1.4 \times 10^{22}$ Nm; intermediate events have $\beta = 0.45-0.64$; $M_{\text{max}} = 1.6 \times 10^{20}-2.8 \times 10^{21}$ Nm; and for deep events $\beta = 0.35-0.67$; $M_{\text{max}} = 5.2 \times 10^{19}-2.0 \times 10^{21}$ Nm. These limits correspond to the 95% confidence area. The curves demonstrate that worldwide earthquake size data are reasonably well-approximated by the gamma distribution. The low accuracy of the maximum moment determination makes it impossible to evaluate $M_{\text{max}}$ for various regions, whereas $\beta$-values can be evaluated for various large seismic provinces. As mentioned above, contrary to the $b$-values, $\beta$-values do not exhibit statistically significant variations for such regions.

The Harvard catalog covers only the last 17 years, a relatively quiet period of global seismicity. During the 1950s and 1960s several great earthquakes occurred, the largest of them being the Chilean earthquake of 1960 with $M \approx 2 \times 10^{23}$ Nm. Thus the value of $M_{\text{max}}$ for shallow earthquakes quoted above, seems to be too low. Seismic moment calculations performed by Kanamori (1977) give us some indication of the possible value of the $M_{\text{max}}$. In Fig. 4b we display the Harvard curve for shallow seismicity together with Kanamori’s data for the period 1921–1976. Curves for the gamma distribution and 95% confidence curves are also shown.

The number of earthquakes in the Kanamori (1977) catalog with $m \geq 8.0$ is small (37), and corresponding likelihood function contours are far from being elliptical. Thus we choose several points on the 95% contours to represent the variability of estimates. The maximum likelihood values are $0.75$ for $\beta$, $M_{\text{max}} \approx 10^{24}$ Nm. A significantly higher value of $M_{\text{max}}$ might be due to the obvious lower accuracy seismic moment determination, to greater statistical variability of these estimates, as well as to possible long-term variations of global seismicity. However, the upper 95% curve for the Harvard catalog yields approximately the same value of $M_{\text{max}}$ as the lower curve for the Kanamori catalog. Therefore, the value of $M_{\text{max}} \approx 3 \times 10^{22}$ Nm ($m_{\text{max}} \approx 9.0$) seems to be an acceptable compromise value. This value of $M_{\text{max}}$ is not an absolute maximum: the gamma distribution allows earthquakes with arbitrarily large moment, but the frequency of these earthquakes drops down exponentially fast for $M \sim M_{\text{max}}$ (see Eq. (3)). We obtain similar results for the Pacheco and Sykes (1992) catalog: for the period 1905–1976 the number of earthquakes with $m \geq 8.0$ is 56, the maximum likelihood values are $\beta \approx 0.95$ and $M_{\text{max}} \approx 10^{24}$ Nm, the lowest acceptable maximum moment is $5 \times 10^{22}$ Nm.

The brittle seismogenic crust layer is relatively thin (about 15–20 km). Consequently one should expect that the size distribution exhibits scaling breakdown (Pacheco and Sykes, 1992; Pacheco, Scholz and Sykes, 1992; Sornette, 1992; Ben-Zion and Rice, 1993; Ben-Zion and Rice, 1994) around $M = 10^{18}-10^{19}$ Nm (the linear size of these earthquakes corresponds to the thickness of the layer). For small earthquakes (with focal region size less than 15 km) the rupture propagates along a 2-D surface in a 3-D space, whereas for large shallow earthquakes the rupture is constrained in a horizontal direction and therefore is concentrated essentially at two points (see Fig. 1a), hence it is 1-D
Fig. 4. Log of scalar seismic moment vs cumulative frequency for the 1977–1993 Harvard catalog. The curves show the numbers of events with the moment larger or equal to $M$. (a) The upper solid curve is for shallow earthquakes (depth 0–70 km). The middle solid curve is for intermediate earthquakes (depth 71–300 km). The lower solid curve is for deep earthquakes (depth 301–700 km). (b) Comparison of moment-frequency relations for shallow earthquakes in the Harvard and Kanamori (1977) catalogs. The approximation by the gamma distribution is shown by a dashed line, dotted lines are curves corresponding to the 95% confidence envelope.
in the 2-D space. However, as seen from Fig. 4a, the size distribution does not display, at least visually, any significant trace of that crossover; in Fig. 4b the curve for shallow earthquakes is within the 95% confidence envelope.

Pacheco et al. (1992) showed that the crossover hypothesis is apparently confirmed by the statistical analysis of the Harvard list of seismic moment tensor solutions. According to them the small and large earthquakes follow a similar power-law distribution but with different values of the exponent: for smaller shallow earthquakes \(5.8 \leq m \leq 7.4\) \(b_1 = 1.0\) (\(\beta = \frac{4}{3}\)) and for larger events \(7.4 \leq m \leq 7.8\) \(b_2 = 1.5\). Intermediate and deep earthquakes also have a break around \(m = 6.4\) (ibid).

However, we see some problems with the Pacheco et al. (1992) statistical analysis: (1) the statistical procedure used by them (the least-squares fitting of the magnitude-frequency plot) grossly underestimates standard errors of \(b\)-values and has other deficiencies in evaluating the parameters of statistical distributions (Vere-Jones, 1988); (2) the data are discretized in magnitude bins with the width 0.1, such quantization of the data is not necessary for seismic moment and might lead to a bias in \(b\) determination; (3) the deviation from linearity in the log-log plot might occur close to the right-hand tail of the distribution and this \(b\)-value change is clearly connected with the finite size of tectonic plates (it is not the crossover due to finite thickness of the seismogenic zone). Moreover, since the publication of Pacheco et al. (1992) additional seismic moment data became available. Thus we decided to carry out statistical analysis of the data to see whether the hypothesis of nonlinearity of earthquake size distribution can be validated with available data. For easy comparison of the results with Pacheco et al. (1992), I convert the seismic moment data into the magnitude, using (2). More details will be provided in L. Knopoff and Y.Y. Kagan (1994, manuscript in preparation).

Analyzing the size distribution we use the results obtained by Deemer and Votaw (1955) for censored and truncated exponential distributions. We test statistically for the possibility that small and large earthquakes belong to different populations. Thus the size distribution for smaller events is truncated at \(m_c\), the crossover magnitude. For larger earthquakes the population is truncated at \(m_x\), maximum magnitude, i.e., we assume that the largest earthquakes again belong to a different population. To avoid the bias connected with the bend due to the largest events, we take \(m_x = 7.8\) which is \(m_{max} - 0.5\) (see above).

Fig. 5a displays curves of the \(b\)-values obtained for shallow earthquakes in the Harvard catalog. We take \(m_c\) changing from 5.8 to \(m_x\) by step 0.1. The values of parameters \(b\) and \(\sigma\) (the standard error in \(b\) evaluation) are then determined for both populations of events by the maximum likelihood method (Deemer and Votaw, 1955). The \(b\)-values and their 90% confidence limits (\(\pm 1.645\sigma\)) are shown as they depend on \(m_c\). The 90% domains of \(b\)-values intersect for all values of magnitudes. Actually for \(m_c\) close to 7.4, \(b_2 < b_1\), i.e., the behavior contrary to the Pacheco et al. (1992) predictions. The inspection of the curve for shallow earthquakes in Fig. 4 explains the result: there is a distribution 'bump' at about \(10^{20}\) Nm and larger, corresponding to the magnitude value of 7.3 (2). The bump is responsible for the dip in \(b_2\) values. However from Fig. 4b the bump is still inside of the 95% confidence interval, thus it does not contradict the gamma distribution (3).

As a statistical test we compare the difference of two \(b\)-values with their standard deviations, \(\sigma\). The ratio

\[
\frac{b_1 - b_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}
\]

is distributed according to the normal distribution with a standard deviation of 1. Since the above expression is valid only for a large number of events, we simulate earthquake sequences.
Fig. 5. $b$-values for shallow earthquakes, $m_c = 7.8$. (a) Upper diagrams show $b$-values and their standard deviation multiplied by 1.645 for earthquake population on the left of the cutoff $m_c$ (dashdot line is $b_1$-value and dotted lines are standard errors), and to the right of $m_c$ (solid line is $b_2$-value and dashed lines are standard errors). (b) Lower diagrams show $b_2 - b_1$ (solid line), and the estimate of the 5% significance levels (dashed line) obtained through simulations.
with $b_1 = b_2 = 1.0$ (the null hypothesis) and appropriate numbers of events, and compute the 5% significance level for these synthetic catalogs. This level is close to that calculated using (4). Fig. 5b displays empirical differences $b_2 - b_1$ and the 5% significance limits. Again the experimental curve is below the 5% significance level. Hence such a curve represents an earthquake sequence satisfying the null hypothesis. Similar results have been obtained for intermediate and deep earthquakes: no statistically significant crossover exists at magnitudes 6.0–7.0.

We also simulated earthquake sequences with the same numbers of events as in the Harvard catalog, with two populations of events having $b_1 = 1.0$ and $b_2 = 1.5$ and then processed them applying similar statistical techniques as with the real data. The simulations show that the quantity of available data is insufficient to resolve the crossover problem, even if a $b$-value change exists (as mentioned above, no evidence is given in the Harvard catalog). Only if the magnitude break occurs in the middle (6.7–7.1) of the magnitude interval available for the statistical analysis (5.8–7.8, see above), we will be able to establish the existence of a crossover for shallow events. With the same type of data it will require significant additional observation time to prove or disprove the existence of $b$-value variation of the magnitude interval 5.8–7.8.

Among the modifications proposed to alter the distribution at its high end, there is the ‘characteristic’ earthquake hypothesis (Scholz, 1990; Nishenko, 1991) which states that although the size distribution of earthquakes for a large area follows the power law, for individual faults or fault segments it has a significantly different form: the largest earthquakes form a separate, independent population (characteristic events). The frequency of the characteristic earthquakes is about an order of magnitude higher than the frequency obtained by the extrapolation of the curve from small events. The consequence of this hypothesis is that almost all of the seismic moment release is contributed by the characteristic earthquakes. This hypothesis has an added significance, since several attempts have been made to obtain a similar distribution using the Burridge–Knopoff (1967) block-spring model of an earthquake fault (Carlson and Langer, 1989; Brown et al., 1991; Shaw, Carlson, and Langer, 1992). Ben-Zion and Rice (1993; 1994) have shown that the characteristic earthquakes can arise in more realistic models, based on continuum mechanics. In their models characteristic-like earthquake behaviour is the result of finite thickness of the seismogenic layer as well as other geometric structures in elastic solid.

It can be shown (Kagan, 1993, and references therein) that the empirical statistical tests of this hypothesis which have been published thus far, have serious defects, and therefore fail to validate the characteristic earthquake hypothesis. We cannot statistically test the characteristic earthquake hypothesis through a retrospective analysis, since current earthquake fault segmentation models are descriptive: hence the number of degrees of freedom in the models is comparable to the number of data. However, it is possible to test the actual forecast of characteristic earthquakes where such considerations do not apply. Nishenko (1991) published a map and a report evaluating the future Circum-Pacific earthquake potential, using one of the versions of the characteristic earthquake hypothesis. The prediction starts on January 1, 1989, and is extended for 5, 10, and 20 years. We compare this prediction with the actual seismicity record as found in two earthquake catalogs: the PDE (1993) list and the Harvard catalog. Only two earthquakes rigorously satisfy the prediction criteria in both of these databases: Costa-Rican earthquake of 1990 ($m_w = 7.3$, in the Harvard catalog), Loma Prieta, California, 1989 earthquake (in both catalogs, $m_w = 6.9$, $m_S = 7.1$), and Landers, California, 1992 earthquake ($m_S = 7.5$, in the PDE). The sum of characteristic earthquake probabilities for five years in Nishenko (1991) is 9.2. The probability of only two or fewer earthquakes occurring, if the predicted rate is 9.2, is
about 0.5%. Even if we accept the Costa-Rican event \((m_S = 7.0)\) and the Japanese earthquake of November 1, 1989 \((m_S = 7.4)\) as satisfying the prediction criteria, the probability is still 4.9%. We have also determined the size distribution of earthquakes in the zones selected by Nishenko (1991): the distribution is inconsistent with the characteristic hypothesis. Instead of a deficit of earthquakes when the magnitude approaches the characteristic limit, earthquake numbers are distributed according to the G–R law. Thus the characteristic hypothesis can be rejected at a high confidence level.

The observed pattern of earthquake size distribution can easily be explained by a simpler assumption of the gamma distribution \((3)\). The appearance of two different populations of events in the computer modelling of earthquake rupture propagation (Carlson and Langer, 1989; Shaw et al., 1992; Brown et al., 1991) is caused, most probably, by the presence of two sets of springs in the Burridge–Knopoff model of earthquake fault. Since the springs have different stiffness, we should expect this model to yield two populations of slip events. In Ben-Zion and Rice’s (1993; 1994) simulations, the characteristic earthquakes appear as the result of certain plausible geometrical patterns introduced into the structure of a tectonic fault. It is to be expected that a particular scale length introduced in a model would manifest itself in the breaking of a scale-invariance. However, since the mechanical models are highly non-unique, the final decision on whether the characteristic hypothesis is correct, belongs to the results of the statistical data analysis, and these results seem to reject the hypothesis. A more detailed statistical analysis of catalog data is needed to solve this problem (cf. Kagan, 1993).

In California we can ask whether the magnitude distribution changes in time. Fig. 6 displays three sets of magnitude-frequency relations normalized to the earthquake rate per year. The catalogs involved are the Ellsworth (1990) list which starts with a 1769 earthquake (the catalog is reasonably complete for \(m \geq 6.0\) since 1850), the Caltech 1932–1993 catalog (Hutton and Jones, 1993, and references therein), and the Harvard catalog. The Pareto relation with \(b = 1\) is also shown in the plot. This \(b\)-value is obtained for the Harvard catalog (see above), for the Caltech and the Ellsworth catalogs which use local magnitude \(-m_L\), \(b = 0.90–0.95\) is more appropriate. Generally, these distributions demonstrate the stability of seismic energy release over the last 140 years. There is rather good agreement between the Ellsworth and Caltech data, the Harvard curve is slightly higher than the other two, implying either a higher rate of seismic activity in the last 16 years, or possibly a systematic bias in comparison of the moment versus the magnitude. The magnitude-frequency plot is linear for the Caltech catalog in the magnitude range 5.0–7.0, thus if the crossover at \(m = 6.0–6.5\) exists (Pacheco et al., 1992), it does not manifest itself in this plot.

The tectonic deformation rate in Southern California is known from geodetic measurements as well as from geologic evaluations of slip on known faults. We can calculate the seismic moment rate, provided that the tectonic deformation is largely expressed through earthquakes. If we assume that the value of \(\beta\) in California is the same as for the whole world, using (3) it is possible to calculate \(M_{\text{max}}\) (Kagan, 1991b; Kagan, 1993, Eq. (14)). We obtain \(M_{\text{max}} = 1.0 \times 10^{21}–2.5 \times 10^{22}\) Nm, i.e., the values which are close to those obtained for the worldwide shallow seismicity (see above).

This result as well as the results reported in Kagan (1991b) suggest that the earthquake size distribution has a universal scale-invariant form which is described by two parameters \(\beta\) and \(M_{\text{max}}\), and the values of these parameters are the same over all seismogenic provinces of the world, with \(M_{\text{max}}\) decreasing for deeper earthquakes. Shallow earthquakes differ from other events by the presence of a large number of aftershocks (catalogs of shallow earthquakes contain up to 30–40% of aftershocks – Kagan, 1991b).
If we take these aftershocks into account and consider the size distribution of earthquake sequences, the value of $\beta$ for shallow seismicity is comparable to that of deeper events, i.e., $\beta \rightarrow 0.5$ (ibid). Thus, the value $1/2$ for $\beta$ may be a universal constant.

4. Spatial pattern – $\mathbb{R}^3$

The fractal spatial structure of rock fracture has attracted many investigators (see, for instance, Lei, Nishizawa, and Kusunose, 1993; Robertson et al., 1994, and references therein). Fractal sets can readily be visualized in this case, hence an intuitive appeal of the measurements is obvious. These studies confirm the spatial self-similarity of fracture and produce several values for the spatial fractal dimension of earthquake fracture (Kagan, 1991a). However, many of these measurements do not relate directly to the earthquake fracture, since they measure the end product of the fracture process: fault traces or fracture rock fragments. These objects can be strongly influenced by decompression (absence of lithostatic pressure) and by the influence of a free boundary (ibid), such as the Earth’s surface. In the measurement of fault patterns on the Earth’s surface or in laboratory specimens (see, for example, Sornette et al., 1993, and references therein) one of the difficulties is the objective tracing of faults: if the faults are fractal, self-similar objects, fault segments can either be combined into larger segments or subdivided into subsegments. Presently there is no objective robust tool for subdividing a fault system into separate entities, thus fault maps are subjective.

Even measurements of the fractal dimension of a set of earthquake hypocenters are subject to various types of systematic and random errors, among the most important are the time span of a catalog, hypocenter location errors, and projection of hypocenters on the Earth’s surface. Failure to take these errors into account would strongly influence the value of the fractal dimension (Kagan, 1991a).

Using several earthquake catalogs we analyze the distribution of distances between pairs of...
earthquake hypocenters to determine the spatial fractal correlation \((D_2\) or \(\delta\)) dimension of an earthquake fracture. As the time span of the catalog increases, \(\delta\) asymptotically reaches the value 2.1–2.2 for shallow earthquakes. Approximately the same asymptotic dimension value is obtained for earthquake catalogs with aftershocks removed. The fractal dimension value declines to 1.8–1.9 for intermediate events and to 1.5–1.6 for deep events. When comparing the values above, one needs to take into account that most of the relative motion between tectonic plates is accounted for by brittle shear deformation caused by shallow earthquakes, whereas for intermediate and deep seismicity, most of the rock deformation is aseismic. For all earthquakes the spatial scale-invariance breaks down for distances between hypocenters of 2000–3000 km (Kagan, 1991a); this distance corresponds to the average size of continents or to thickness of the mantle.

To obtain a better understanding of the spatial structure of earthquake faults we have also determined three-, and four-point moment functions of the spatial distribution of shallow earthquake hypocenters (Kagan, 1982). After the non-uniformity of the depth distribution of hypocenters is taken into account, the results once again indicate lack of any preferred scale of distance or configuration size in these correlation functions. Let \(S\) and \(V\) be the surface area of a triangle and the volume of a tetrahedron, formed by lines connecting 3 and 4 hypocenters respectively. The probability of finding a triangular or tetrahedral correlation in catalogs of shallow earthquakes is approximately proportional to \(1/S\) and \(1/V\) in order of decreasing degree of certainty. There is sufficient evidence that in the three-point case this probability is independent of the form of the triangle once the surface is fixed. A similar effect for the four-point function is also indicated, though less convincingly. The results for the three-point function are consistent with the standard model (see Section 2) that treats an earthquake fault as an isolated plane. However, if hypocenters are constrained to a plane, the distribution of tetrahedral volumes would be a delta function. Since the distribution is instead proportional to \(1/V\), the results for the four-point case contradict a planar fault model (Kagan, 1982).

The two-, three-, and four-point moments are special cases of general multifractal approach for studying the spatial distribution of earthquake foci (Lei et al., 1993; Robertson et al., 1994, and references therein). I have little doubt that the application of multifractal techniques to earthquake data is, in principle, superior to the classical fractal analysis. However, there are many difficulties with the application of multifractal methods. For example, the spatial distribution of earthquakes is influenced by many nuisance factors, mentioned above. This 'noise' is largely responsible for the fact that there is no consensus on whether, for instance, the hypocenter correlation dimension has a worldwide universal value, or if it changes with time and seismogenic region (as mentioned in Section 3, similar problems exist for the \(b\)-value). If we cannot take into account these factors for even the simplest case of the correlation dimension, what is the guarantee that the multifractal results are not artifacts of all the noise described above (cf. Eneva, 1994)?

There have been relatively few attempts to reproduce the scale-invariant spatial property of the fracture in simulations or in laboratory experiments: Ito and Matsuzaki (1990) obtained \(\delta = 1.1\) in a 2-D, Herrmann and Roux's (1990) book supplies several other examples. Hirata, Satoh, and Ito (1987) investigated the distribution of microcrack foci in rock specimens and found it to be self-similar with the fractal dimension decreasing from 2.75 to 2.25 during the process of specimen failure under uniaxial pressure. Robertson et al. (1994) and Sahimi, Robertson, and Sammis (1993) find the value of \(D_0\) (the fractal capacity dimension) in 3-D is less than 2.0 for several small regions in California. They explain these low dimension values by the fact that earthquakes occur only on a percolation
Our measurements of \( \delta \) have been made using both the worldwide and local catalogs. In global catalog evaluations the boundary effects disappear. Moreover, both spatial and temporal variability of the estimates is suppressed due to the relatively long-term span of the catalogs averaged over many tectonic provinces. The \( \delta \)-values obtained for local catalogs usually display much greater variations due to random fluctuations of seismicity. For example, the Southern California (Caltech) catalog is dominated by three earthquake sequences (Kern County, 1952; San Fernando, 1971; and Landers, 1992, earthquakes), and it is difficult to find a statistical technique which suppresses the prevalent influence of these sequences on the results of an analysis. However, in the worldwide database, there are hundreds of similar earthquakes, thus the influence of individual sequences is greatly diminished.

5. Time distribution - \( T \)

Omori (1895) and several other researchers have shown that the frequency of an aftershock occurrence decays in time \( (T) \) as \( T^{-p} \), where the exponent \( p \) is slightly larger than 1.0 (Kagan and Jackson, 1991a). Foreshocks have been found to follow a similar pattern, their numbers increasing as a power law with a similar value for the exponent, as the mainshock approaches. Similar scale-invariant distribution of aftershocks is found for acoustic emission during microfracturing of rock specimens (Hirata, 1987).

There are several problems involved in the statistical study of temporal earthquake patterns: instability of statistical estimates for a decay exponent due to general difficulties of statistical analysis of power-law (or stable) distributions (Zolotarev, 1986). Moreover, straightforward investigation of individual aftershock sequences is less reliable, since samples are biased by selection (only large aftershock sequences can be studied, whereas even many strong earthquakes do not produce such sequences) and because of secondary clustering of aftershocks. Attempts to determine the fractal dimension of the temporal projection of earthquake sequences have not been completely successful (Kagan and Jackson, 1991a), since the dimension depends on the degree of spatial averaging.

Therefore, we attempt to study the temporal behavior of earthquakes by simulating the sequences, processing them through the same maximum likelihood procedure that is applied to natural earthquakes, and then comparing both types of results (Kagan and Knopoff, 1981). In our simulations we allow elementary seismic events to develop according to a critical branching process: the probability of each event producing another event in a time interval \( dt \) is proportional to \( t^{-1-\theta}dt \). The graphs of cumulative seismic moment produced by this model clearly show the hierarchical clustering patterns. The source-time function appears to be composed of several steps, each starting with a sharp onset and usually becoming more gradual later. Thus the sequence has few foreshocks and many aftershocks, as do natural earthquake sequences. By its construction, a simulated earthquake sequence is an asymmetric Cantor set.

Processing the simulated earthquake sequences by the maximum likelihood procedure we find that the value of \( \theta = 0.5 \) generates sequences which have the temporal properties of shallow earthquakes: they have foreshocks and aftershocks, and the numbers of these dependent events are the same as for natural earthquake catalogs. The value of \( \theta = 0.8-0.9 \) produces sequences with few if any aftershocks; the almost total absence of aftershocks is the property of intermediate and deep earthquakes. All earthquakes have a power-law size distribution with the value of \( \beta \) about 0.6-0.7 for shallow earthquakes and \( \beta = 0.5-0.6 \) for deeper events (Kagan and Knopoff, 1981).

However, it is more difficult to study long-term properties of an earthquake occurrence: available catalogs are usually too short when
compared with the recurrence time of the largest events (decades and centuries). There have been continuing attempts in seismology to prove that earthquakes, especially very large ones, are periodic or quasi-periodic. Nishenko and Buland (1987) analyzed about 50 pairs of earthquakes which have occurred during the last several hundred years, postulating that the inter-earthquake time is distributed according to the log-normal distribution with a relatively small coefficient of variation (0.21). Their study seems to be confirmed by the simulation of earthquake sequences using the Burridge-Knopoff model (Carlson and Langer, 1989; Huang and Turcotte, 1990; Brown et al., 1991; Huang, Narkounskaiia, and Turcotte, 1992): in these investigations synthetic sequences of the strongest events are also quasi-periodic.

However, claims of earthquake quasi-periodicity should be treated with great caution: if we calculate the total number of events available in principle for analysis by Nishenko and Buland (1987) – magnitude 6 and larger events which occurred during the last few hundred years – the number is of the order of tens of thousands (see Fig. 4). Of course, not all earthquakes have been recorded in historical, instrumental, and other catalogs, but available records nevertheless contain thousands of such events. A possibility for biased sampling of the phenomenon which exhibits a great degree of randomness, is very strong, hence it is very important that the events selected for statistical study be representative of the general behavior of earthquakes. Kagan and Jackson (1991a) analyze all earthquakes, available in the instrumental catalogs: the coefficient of variation for inter-earthquake times is consistently higher than 1.0, therefore we conclude that the most important feature of seismicity is long-term clustering. In another paper Kagan and Jackson (1991b) test whether the prediction made 10–15 years ago and based on the model of earthquake quasi-periodicity (the seismic gap model), has been validated by the history of recent large earthquakes.

The results of the statistical hypothesis test is that the antithesis of the quasi-periodic hypothesis (the temporal clustering of events) fits the data better than the gap hypothesis. However, the null hypothesis (the Poisson process) cannot be rejected for any of the catalogs.

Kagan and Jackson (1991a, 1991b) suggest that even after strong earthquakes which rupture the whole Earth’s crust, the numbers of equally strong events decay in time according to a power law, contrary to the expectations of the ‘periodic’ hypothesis. These results are consistent with a hypothesis that tectonic stress is always high and close to the critical value, even after a large earthquake, as proposed by the theory of “self-organized criticality” (Bak and Chen, 1991, and references therein; Olami and Christensen, 1992).

In the Burridge-Knopoff model a large slip event releases most of the elastic energy, which in this model is stored only in the springs. A new large event is possible only after a significant period of time of stress accumulation has elapsed. In the Earth, the potential elastic energy may be stored in tectonic plates or in the mantle which measures thousands of km, therefore even the largest earthquakes may release only a
small part of the energy. Moreover, a fracture depends, in principle, on three stress tensor invariants (Kagan, 1990); stress variations caused by large earthquakes may modify the stress pattern and therefore an occurrence of subsequent events in a very complex manner.

To study the long-term properties of seismicity, we statistically analyze (Kagan and Jackson, 1991a) several instrumental earthquake catalogs. After we take into account the effect of short-term clustering (aftershocks), the degree of clustering in residual catalogs is the same for earthquakes in different depth ranges. Therefore, we conclude that time clustering of earthquakes is a universal phenomenon which has two general features: (1) a short-term, strong clustering of shallow earthquakes responsible for foreshock-mainshock-aftershock sequences, and (2) long-term, weak clustering which characterizes all mainshock earthquakes – shallow, intermediate, and deep. There is circumstantial evidence that long-term variations of seismicity similar to a short-term clustering, are governed by a power-law temporal distribution, i.e., they are fractal. The fractal dimension of the set of earthquakes on the time axis is of the order of 0.8 to 0.9, the mainshock occurrence is much closer to a stationary Poisson process than standard aftershock sequences of shallow earthquakes. Thus we conclude that both short- and long-term earthquake behavior can be explained by fractal distributions with the value of exponent \( \theta = 0.5 \) for shallow events and \( \theta = 0.8-0.9 \) for deep earthquakes.

6. Rotation – SO(3)

Kagan (1982) introduced the rotational Cauchy distribution (see below) to represent rotations of focal mechanisms of microdislocations which comprise the focal zone of an earthquake. The Cauchy distribution is especially important for representation of earthquake geometry since it can be shown by theoretical arguments (Zolotarev, 1986, pp. 45–46; Kagan, 1990) and by simulations (Kagan, 1990) that the stress tensor in the medium with defects follows this distribution. For any point in an elastic medium which is surrounded by defects, the characteristic function for the random stress distribution can be written as

\[
\log \phi(\zeta, \alpha) = \int_0^\infty \left[ \exp(i\zeta \sigma r^{-3}) - 1 \right] \nu(r) r^2 dr
\]

where \( \nu(r) \) is the density of defects which might depend on \( r \), distance of a defect from the reference (measurement) point, and \( \sigma \) is the normalized (for \( r = 1 \)) stress Green function of an earthquake; stress decays with distance as \( r^{-3} \). To simplify the notation, we display the distribution for only one component of a stress tensor; the distribution for any other component differs only by a scale factor. For the uniform 3-D distribution of defects, \( \nu = \nu_0 \). In this case (5) yields the Cauchy distribution (see also Eq. (8) below).

It is difficult to measure the stress tensor itself in the deep interior of the Earth, but rotations of earthquake focal mechanisms give us an indication of the stress redistribution. We argue (ibid) that the Cauchy distribution of the stress should produce the rotational Cauchy distribution of earthquake sources.

The Cauchy law is a stable distribution (Mandelbrot, 1983; Zolotarev, 1986; Takayasu, 1990). The stable distributions are important for two reasons: (a) They are invariant under addition of random variables – suppose that two independent stochastic variables \( X_1 \) and \( X_2 \) are distributed according to the one-parameter Cauchy distribution with the parameter values \( \kappa_1 \) and \( \kappa_2 \). The distribution of their sum, \( X \), is a convolution of the two distributions

\[
F_\kappa(X) = F_{\kappa_1}(X_1) * F_{\kappa_2}(X_2),
\]

where \( \kappa = \kappa_1 + \kappa_2 \). This invariance under addition is the reason the distributions are called stable (Mandelbrot, 1983, p. 367ff). (b) The stable
distributions have a power-law tail, i.e., they are scale-invariant.

The general rotational Cauchy distribution can be written as (Kagan 1982; 1990)

\[ F(\Phi) = \frac{2}{\pi} \left[ \arctan(A/\kappa) - \frac{A\kappa}{A^2 + \kappa^2} \right], \tag{7} \]

where \( A = \tan(\Phi/2) \) and \( \Phi \) is the rotation angle. The scale parameter \( \kappa \) of the Cauchy distribution represents the degree of incoherence or complexity of an earthquake fault.

An additional complication in the study of the 3-D rotation of earthquake focal mechanisms is the symmetry of the source: the double-couple earthquake source has the rotational symmetry of a rectangular box with unequal sides (the dihedral group \( D_2 \)). Due to this symmetry, the maximum rotation angle for the earthquake source cannot exceed 120° (Kagan, 1990).

Using the correspondence between the group \( SO(3) \) and the group of normalized quaternions, we have solved an inverse problem of a 3-D rotation of double-couple earthquake sources, i.e., for each pair of focal mechanisms we find a minimum 3-D rotation which rotates one mechanism into another (Kagan, 1992b). In Fig. 7a we display histograms for the distribution of rotation angle \( \Phi \) for shallow earthquake pairs which are separated by a distance of less than 50 km. We study whether the rotation of focal mechanisms depends on the location of the second earthquake of the pair with regard to the first event. Thus we measure the rotation angle for hypocenters located in 30° cones around each principal axis (curves marked the T-, P-, and N-axes) of the first event (see Figs. 1 and 2). The curves in Fig. 7a are narrowly clustered, and are obviously well-approximated by the rotational Cauchy distribution.

Fig. 7b shows \( \Phi \)-histograms for shallow earthquakes separated by 400–500 km. Here the curves corresponding to fault-planes (the N-axis) are clearly separated from the histograms connected with the T- and P-axes: whereas the rotation near the fault-plane is relatively small (\( \kappa \approx 0.2 \)), the earthquakes which are situated in cones around the T- and P-axes have focal mechanisms which are essentially uncorrelated with the primary event: the curves are close to the curve corresponding to completely random rotation of a double-couple (Kagan, 1990; 1992b). We obtain similar results for intermediate and deep earthquakes.

7. Incremental stress

In the previous section we analyzed the rotation of earthquake focal mechanisms. These rotations are influenced by the patterns of total stress tensor accumulated during a very long-term evolution of an earthquake focal zone. The total stress is the combination of lithostatic stress, tectonic regional stress, and static stress changes during earthquakes. The last stress ingredient (incremental stress) can easily be evaluated from modern earthquake catalogs. However, using an instrumental catalog, we observe only a short time interval in the development of earthquake fault zones whose history is sometimes measured in millions of years.

The earthquake spatial distribution, as described in Section 4, is fractal. Its fractal dimension increases with the time span of an earthquake catalog. In Eq. (5) we should substitute the fractal distribution of sources \( \nu = \nu_0 r^{D-3} \), where \( D = 3 \) is the Euclidean dimension of the space, and \( \delta \) is a fractal correlation dimension of earthquake hypocenters (Kagan and Knopoff, 1980; Kagan 1991). Then (cf. Zolotarev, 1986, Eq. (1.1.16))

\[
\log \phi(\zeta, \alpha) = \nu_0 \int_0^\infty [\exp(i\zeta u) - 1] u^{(\delta/3) - 1} du \\
= \nu_0 \Gamma(-\alpha)|\zeta|^\alpha, \tag{8}
\]

with \( \alpha = \delta/3 \). \( \Gamma \) in (8) is the gamma function. The above formula means that if \( \delta = 3 \), the resulting distribution is the Cauchy law.
Fig. 7. Distributions of rotation angles for pairs of focal mechanisms of shallow earthquakes from the Harvard catalog; hypocenters are separated by distances (a) 0–50 km; (b) 400–500 km; circles – hypocenters in 30° cones around the T-axis; plusses – hypocenters in 30° cones around the P-axis; stars – hypocenters in 30° cones around the N-axis. Solid line is for the Cauchy rotation with (a) $\kappa = 0.1$; (b) $\kappa = 0.2$; dashed line is for the random rotation.
(Zolotarev, 1986; Kagan, 1990), whereas for a fractal spatial distribution of earthquakes $\alpha < 1$.

In Fig. 8, we show several curves calculated for stress components $S = (|S_{11}| + |S_{12}| + |S_{22}|)/3$. These components have been selected since the horizontal components are larger than the vertical ones for shallow earthquakes (Kagan, 1994a). We use the sum of three components to suppress random fluctuations. The sum of stable variables is distributed according to the same stable distribution, as a single variable.

We calculate the incremental stress at the centroid point of each of the reference points for several subcatalogs: one curve is for the full Harvard list, i.e., for 10,566 earthquakes registered over 16 years, other curves are for the stress caused by earthquakes which are separated from a reference event by no more than 1250, 250, and 50 catalog entries (see more detail in Kagan, 1994b). This means that on an average we compute the stress caused by earthquakes in time intervals not exceeding 16, 1.9, 0.4, and 0.08 years.

Several possible sources of error need to be considered with regard to the experimental data. First, earthquake locations are greatly perturbed by location errors which randomize the position of earthquake centroids. The error is especially serious when distances between earthquakes are small. The location errors for modern worldwide earthquake catalogs are of the order of 10 km (Dziewonski et al., 1993). Second, the point model for an earthquake source is inappropriate for small distances. During a large earthquake, the rupture propagates over a plane-like volume of significant extent. Even if we use the Green function for an extended fault, the computed resulting stress pattern is a gross over-simplification. Shallow earthquakes are accompanied by a large number of aftershocks which are concentrated in a focal zone, manifesting large stress concentrations inside the zone. The measurements of surface displacement in the recent Landers earthquake (Sieh et al., 1993) demonstrate that the earthquake slip changes drastically over small fault distances. Hence there are very large residual stresses which remain after an earthquake.

I calculate the theoretical stable distributions using the formulas from Zolotarev (1986) or tables by Holt and Crow (1973). When comparing the theoretical and experimental curves, we can vary the values of two adjustable parameters, that of $\alpha$ (index of a distribution) and of $\lambda$ (scale factor). The change of $\lambda$ causes a horizontal shift of theoretical curves in the diagrams in Fig. 8, whereas a modification of $\alpha$ leads to a change of the asymptotic slope of the curves for $x \to \infty$ (distribution tail), as well as to a change of the curvature. Since the $\lambda$ parameter has not been evaluated and adjusted in Fig. 8, the curves can be arbitrarily shifted in a horizontal direction. For all the experimental curves, the distribution of stress for large abscissa values has a slope approaching one, hence they are closer to the Cauchy distribution. These large stresses are caused by near earthquakes, and the hypocenters of these earthquakes have a fractal dimension close to three due to location errors (Kagan and Knopoff, 1980; Kagan, 1991). Thus the slope of the curves at the right-hand tail of the distribution is an artifact of location errors. If this assumption is true, the curves should be extrapolated from the values in the $10^{-2}$-$10^{-1}$ bar interval into the tail area.

The experimental curves for small stress values (less than 1 bar) are less influenced by the location and errors discussed above. For stress less than 1 bar, the experimental curves for large time intervals are approximated by stable distributions with $\alpha \approx 0.5-0.7$, and for smaller time intervals the $\alpha$ value approaches 0.25, possibly less. The curves for a small time span seem to have a power-law type of dependence (linear in the log-log plot) in the stress interval of the $10^{-2}$-$10^{-1}$ bar. The earthquake fractal (correlation) dimension is dependent on the time span ($\Delta T$) of a catalog: for example, for a one day interval between earthquakes the dimension is $\delta \approx 1.0$; for 0.1 year interval $\delta \approx 1.5$; and for one year interval $\delta \approx 1.9$ (Kagan, 1991, Tables 2 and 3).
According to (8) $\alpha = \delta / 3$. Thus the predicted values of $\alpha$ exponent are similar to those obtained through the fitting of the curves to theoretical distributions (Fig. 8).

A commonly accepted model suggests that earthquake triggering is largely controlled by the Coulomb failure stress changes $s_f$ (Jaeger and Cook, 1979; Scholz, 1990): $s_f = \tau + \mu s_n$, where $\tau$ is the incremental shear stress on a fault plane, $\mu$ is a static (positive) coefficient of friction, and $s_n$ is a normal stress change on the plane (positive $s_n$ corresponds to relative extension). Laboratory experiments (ibid) yield the value of $\mu = 0.7$–0.8 for most rock materials. Therefore, an earthquake would be more likely to occur when $\tau + \mu s_n \geq 0$.

I have calculated $\tau$ and $s_n$ on both nodal planes for centroid points of each earthquake in the Harvard catalog (see Fig. 9 for the distributions). The accumulated stress values are computed at the location of any future reference earthquake (pre-stress) and then compared to the stress level measured at the same point after the event (post-stress). Fig. 9a displays the histogram for the $s_n$, normal resolved stress, for one of the nodal planes. The distribution is similar (Kagan, 1994a) for the other plane. The plot implies that the friction coefficient $\mu$ is small. We would expect the distribution of the normal stress $s_n$ at the place of a future earthquake to be highly asymmetric with more earthquakes occurring when the incremental stress is dilatational. Contrary to such expectations, the plot (Fig. 9a) is almost symmetric with regard to zero. The inspection of cumulative curves suggests that the distribution is slightly asymmetric. However, curves for various seismic regions and curves corresponding to a finer subdivision of the depth intervals show that this asymmetry is not consistent over various seismic provinces or depth intervals (Kagan, 1994a).

Asymmetry of pre- or post-stress distributions (Fig. 9b) is much more pronounced for shear stress than is the similar plot for normal stress.
Fig. 9. Distributions of stress resolved on nodal planes for shallow earthquakes in the Harvard catalog. In all plots, the solid line is for the stress measured before an occurrence of an earthquake, and the dashed line is for the stress measured after an occurrence of an earthquake. (a) Distribution density for the normal stress $\sigma_{rr}$. Dotted line is for the Cauchy distribution (see text). (b) Distribution density for the shear pre- and post-stress $\tau$. Dotted line is for the Cauchy distribution. (c) Difference between cumulative distributions for the shear pre- and post-stress $\tau$. (d) Difference between cumulative distributions for negative and positive values of the shear pre- and post-stress $\tau$. The absolute value of the stress is the abscissa of the plot.
Fig. 9—continued.
(Fig. 9a). The positive τ corresponds to the stress change which has the same sign as the seismic moment tensor of the reference earthquake. Thus, earthquakes are more likely to be induced by the incremental shear stress if the stress and the moment tensor of the ensuing event are coherent.

To better demonstrate the subtle asymmetries of stress distributions, Figs. 9c,d show two kinds of distribution differences: pre- vs post-stress (9c), and negative values vs positive values of the shear stress (9d). The pre-/post-stress difference is asymmetric: the difference for positive τ is significantly larger than the difference for the negative τ. Nevertheless some events are inconsistent with the incremental shear stress (left-hand side of Fig. 9c). Complex geometry of earthquake faults and errors in a centroid location and in seismic moment inversions might also contribute to the lack of coherence. In plot 9d, we calculate \(1.0 - F(-a) - F(a)\), where \(F(a)\) is the value of a cumulative function corresponding to stress level \(a\). The distribution difference between the positive and negative values of stress reaches 20%, and the difference is smaller by a factor of about 2.0 for the post-stress compared to the pre-stress.

8. Discussion

In the introduction I emphasize the major dilemma for the interpretation of the results of empirical analysis of seismicity: whether the earthquake occurrence is governed by universal scale-invariant distributions, or whether there is a multiplicity of intrinsic scales in space, time, and earthquake magnitudes. I assume that the results presented in the previous sections testify that the former alternative is true: the statistical distributions have a simple underlying form, the apparent diversity of results is due to statistical artifacts or to a short time-span of available earthquake data. What consequences can be drawn from these results with regard to modelling seismicity and understanding the physics of earthquake rupture?

Let us first summarize the results. In Section 3 I argue that the value 1/2 for \(\beta\) – the parameter which controls the earthquake size distribution – may be a universal constant. The earthquake size distribution is the least informative of all the power laws that govern an earthquake occurrence: percolation models with the lattice dimension \(d \geq 6\); self-organized criticality models with \(d \geq 4\) (Chen, Bak, and Obukhov, 1991); a critical branching process (Vere-Jones, 1976) – all yield a power-law cluster size distribution with \(\beta = 0.5\) (Kagan, 1991b). Thus the power-law size distribution can be obtained from a variety of models, and the fact that some events in a model follow a fractal distribution, even with a correct exponent value, does not prove that the model can be used to describe seismicity.

In Section 4 I discuss the measurements of the correlation fractal dimension \(\delta\) for a set of earthquake hypocenters or centroids. Taking into account various possible systematic and random errors, we conclude (Kagan, 1991a, and references therein) that the correlation fractal dimension of brittle shear fracture of rocks is \(2.20 \pm 0.05\). This dimension is smaller for intermediate and deep events, since these earthquakes represent only a small part of rock deformation at great depth. The results appear to rule out the conventional model of earthquake hypocenters occurring on a single isolated plane or on several planes and demand instead that a fault zone for shallow earthquakes be non-planar and fractal. Inhomogeneity of earthquake distribution with depth as well as dependence of fractal dimension on time span of a catalog and other factors makes the dimension determination a difficult problem which still resists a rigorous statistical analysis. Thus the comparison of results obtained for various catalogs or for various dimensions leads to ambiguous conclusions.

Although the earthquake temporal correlation is one of the earliest known examples of the power-law distribution in physics (Omori,
empirical results are still far from being non-controversial. The power law form of short-term earthquake clustering is no longer in doubt, but the value of the exponent has not been agreed upon. Kagan and Knopoff (1981) presented arguments in favor of the $\theta$-value being 1/2 for shallow earthquakes, increasing to 0.8–0.9 for intermediate and deep events. As argued in Section 5, these $\theta$-values do not only explain the short-term temporal behaviour of earthquake sequences, but reproduce the earthquake size distribution, as well as proper rates of foreshocks and aftershocks. The situation is more difficult with regard to long-term earthquake recurrence. Although many seismologists believe that large earthquakes are quasi-periodic, we present evidence that fractal clustering governs long-term earthquake properties, but the value of the exponent ($\theta$) is closer to one in this case, yielding more drawn out earthquake sequences.

The power-law temporal dependence may have a simple explanation: if we assume that stresses at the end of an earthquake rupture are below the critical value and thereafter change randomly according to a one-dimensional Brownian motion, then the level-set of this motion is a fractal set with a dimension 0.5 (Mandelbrot, 1983). Therefore the stress again reaches the critical level at the time moments distributed according to a power law (Kagan and Knopoff, 1987). Then short-term clustering of earthquakes is due to stress diffusion. A tentative interpretation of our results follows: since long-term clustering is a property of earthquakes belonging to all depth ranges, the clustering is due to dynamic processes in the Earth’s mantle. As a conjecture we propose that the value of fractal dimension $\theta$ equal to 0.5 corresponds to temporal clustering of events during a fracture of brittle solid materials. This value of $\theta$ yields a discontinuous release of seismic energy (Kagan and Knopoff, 1981), usually interpreted as fore-, main-, and aftershock sequences. The value $\theta$ between 0.5 and 1.0, on the other hand, should correspond to a plastic flow of materials and instabilities accompanying that flow.

The study of the rotation of earthquake focal mechanisms and incremental stress caused by earthquakes (Sections 6 and 7) began only a few years ago. Research in both of these investigation fields goes to the heart of an earthquake occurrence problem – interaction between the stress and earthquakes. Fortunately, theoretical considerations can be of help in these studies: it can be shown that the stress distribution, under rather general assumptions, should follow one of the symmetric stable distributions with the exponent values $\alpha \leq 1.0$. The observational evidence confirms these theoretical predictions: that the rotations of earthquake focal mechanisms which are caused presumably by stress perturbations, follow the rotational Cauchy distribution, whereas the incremental stress distribution is also governed by stable distributions. The stable stress distributions are a necessary consequence of the presence of defects in rock material. Since a stable distribution has a power-law tail, the stable distributions’ control of stress indicates that the earthquake rupture in presence of defects has to produce the fractal geometry.

Is the fractal dimension of a set of earthquake hypocenters in any way connected with rotation results? An ideal solid crystal (without defects) should fail along a planar dislocation, hence the dimension of fracture should be 2.0; the Cauchy distribution (7) has $\kappa = 0$ for such a crystal. However, the results of our measurements of the focal mechanism rotation indicate that due to the tectonic evolution, focal zones of earthquakes contain partially incoherent defects distributed according to the Cauchy distribution with $\kappa$ as large as 0.2. It would be of great interest to know whether $\kappa = 0.2$ implies the hypocentral fractal dimension $\delta$ of 2.2 (see above). Several different models of earthquake rupture and occurrence have been proposed. The classic model of earthquake source (Burridge and Knopoff, 1967; Aki and Richards, 1980; Scholz, 1990) is a motion of two rigid blocks separated by a single smooth, usually planar,
The resistance of rocks to the motion is described as a static and dynamic friction. The Burridge–Knopoff model was a significant first step in quantitative analysis of seismicity: with very simple initial assumptions the model reproduces some obvious features of earthquake occurrence, i.e., ‘stick-slip’ behavior and rupture propagation. Recently this model and its modifications have been used extensively (Carlson and Langer, 1989; Huang and Turcotte, 1990; Brown, Scholz, and Rundle, 1991; Shaw et al., 1992; Narkounskaya and Turcotte, 1992; Huang et al., 1992; Sornette, Vanneste, and Knopoff, 1992; Vieira, Vasconcelos, and Nagel, 1993) to explore the size distribution of simulated earthquakes, temporal features of synthetic sequences, and chaotic behavior of seismicity, etc.

There are many other computer models of an earthquake occurrence; a relatively comprehensive review is given in books by Herrmann and Roux (1990) and Takayasu (1990). Again one feels overwhelmed by the diversity and multiplicity of the models, of the results of simulations, and by the lack of general physical understanding of the models. Some of the models, like percolation, neglect to take into account the mechanical properties of the medium. Other models also use non-physical or unrealistic assumptions, simplistic geometries of earthquake faults, of the stress tensor, or of medium structure. Without a substantial theoretical base it is difficult to evaluate the results of these simulations and to judge their relationship to earthquake fracture (cf. Kadanoff, 1986).

Almost all of the models above have serious deficiencies, due to over-simplification of the real mechanics of the Earth’s crust: (a) the earthquake fault geometry is imposed from the beginning and is fixed, and not defined by a natural evolution of the model; (b) most models are 1-D or 2-D, thus the 3-D rotation of earthquake focal mechanisms cannot be properly taken into account; (c) as a result of the factors listed above, the stress is taken to be a scalar quantity, thus the full complexity of earthquake behaviour cannot be modelled; (d) the mechanical interaction of model elements is represented as a static and dynamic friction. Below I discuss these drawbacks in detail.

As the results in Sections 6 and 7 suggest, the complexity of real earthquake faults increases in time, with the faults undergoing an evolution, thus the earthquake deformation is time-irreversible. Models which yield a stationary time series of earthquake occurrence lack an important ingredient of fault system development. Therefore long-term predictions made on the basis of these simulations, might be incomplete. The planar faults are geometrically and mechanically reversible in time, i.e., the blocks can be returned to the original position by the opposite motion, implying a complete or almost complete replicability of the process.

One may argue that although the fault development is not stationary, on a geologically short-term scale, it can be approximated by a stationary series. The complexity of an earthquake fault zone is then represented as a spatially variable, complicated friction law. However, anyone who tried to force open a jammed door knows that a miniscule (of the order $10^{-3}$–$10^{-4}$) change of the ‘system’ geometry can modify its mechanical behaviour in a substantial way. During each earthquake the strain drop is of an order $10^{-4}$ (Scholz, 1990), thus the mechanical properties of the fault system should change after each earthquake: new barriers or asperities (disclinations) form as result of an earthquake deformation, and these new complexities modify the fault future behaviour in a way that is unpredictable by simple models of earthquake quasi-periodicity.

The results reported above make us question the suitability of some notions commonly held in the theory of an earthquake source. In particular, I believe that the standard source models are based on the mechanics of man-made objects. Three clearly defined geometrical scales can be distinguished in such objects: (a) exte-
rior or macroscopic, which applies to the whole body; (b) interior or microscopic, which describes defects and intergranular boundaries of the object material; and (c) unavoidable irregularities of the surface of the object. The factors connected with the latter scale are usually taken into account by the introduction of frictional forces (Scholz, 1990). In many engineering applications, great efforts are expended to keep the friction under control and reduce the surface roughness. Despite these measures, surface irregularities of sliding objects always increase, causing the very existence of the separate scales (a) and (c) to be unstable. (This is an example of lack of time-reversibility of the friction.)

Our study of the earthquake spatial statistical properties (Section 4) indicates that there are no separate scales for this process: the process is self-similar and, as such, lacks any intrinsic scale. Moreover, fault geometry analysis (Kagan 1982; 1991a) indicates that earthquakes do not occur on a single (possibly wrinkled or even fractal) surface, but on a fractal structure of many closely correlated faults. The total number of infinitesimal faults can be infinite.

Similarly, scales (a) and (b) are usually separated in materials science and in engineering applications by introducing effective properties of the material. These properties can either be measured in special tests or calculated using empirical engineering formulas. In earthquake studies we consider the propagation of a rupture through rock material which, during millions of years of its tectonic history, has been subjected to repeated earthquake deformation. As a result of this process, the largest defects which we identify with the fault systems, have approximately the same size as tectonic blocks, hence no scale separation is possible. These large defects are critically self-organized (Bak and Chen, 1991; Chen et al., 1991) coherent aggregates of many small earthquake sources.

Fig. 10 shows diagrams for several geometrical/mechanical models of earthquake fault. The friction forces can easily be attributed to the first two models. However, even for a fractal single surface it is not clear how a large displacement can occur; sooner or later a large piece of a block will break off, and the initial geometrical pattern will change. For the last model, the friction is not applicable at all, since the displacement is carried out through many surface-like volumes. Similarly, the time reversibility of faults is different for these models: the first two models are time-reversible (however, see the remark on friction reversibility above); i.e., by inverting the blocks' directions one can revert the system to its initial state. This is not the case with the last two models: large deformation is obviously not reversible.

Almost all of the models above use a quasi-static case, i.e., inertial effects are disregarded. However, during an earthquake, a rupture propagates with a velocity close to the velocity of shear waves. Modelling of the dynamic case is an immensely difficult task, and only for simple systems, like the Burridge-Knopoff model, is full dynamic consideration possible. However, the available evidence indicates that observed
earthquake behavior can be extrapolated toward smaller time and distance intervals. Therefore, we expect that the dynamic modelling of rupture in complex materials with defects will yield scale-invariant features. Indeed, the linear (and nonlinear) elasticity equations do not have any characteristic temporal or spatial scales (however, they do have characteristic velocities: \( V_p \) and \( V_s \), velocities of the longitudinal, \( P \), and shear, \( S \), waves). Thus the distribution of physical quantities during the earthquake rupture process is scale-invariant: we should be unable to judge whether measurements are made with the scales, for example, 1 s and 1 km vs 1 ms and 1 m.

The over-simplification of an earthquake source model may be the cause of the paucity of modelling results compared with observational evidence. Most of the models reproduce, at least approximately, the earthquake size distribution. However, upon closer examination it turns out that either the scaling range is too narrow to be credible (Lomnitz-Adler, 1993), or the value of the scaling exponent does not correspond to that of real earthquakes. Some of the models display a ‘tunable’ \( \beta \)-value which depends on the model parameters (cf. Christensen and Olami, 1992). However, the results in Section 3 suggest that \( \beta \) has a universal value, thus such a tunable model should explain whether \( \beta = 0.5 \) has a special meaning in the model. Moreover, as mentioned above, the \( \beta \)-value is not a very useful parameter.

Attempts to reproduce the Omori law of aftershock rate decay by computer simulations are usually based either on a special mechanism for temporal scale-invariance (Ito and Matsuzaki, 1990) or on the introduction of time delay into a mechanical model by using additional hypothetical assumptions. It is worth mentioning that almost all simulations of earthquake sequences using the Burridge-Knopoff model or other mechanical models (Carlson and Langer, 1989; Shaw et al., 1992; Brown et al., 1991; Herrmann and Roux, 1990; Huang et al., 1992; Chen et al., 1991; Olami and Christensen, 1992) fail to reproduce this most obvious temporal feature of an earthquake occurrence.

The only computer model that I am aware of, which reproduces the major properties of an earthquake process was developed by Mora (1994, see also Donzé, Mora, and Magnier, 1994). This model, consisting of an array of solid lattice particles, realistically reproduces such features of earthquake occurrence as dynamic rupture in a potentially 3-D solid with the propagation of the \( P \)- and \( S \)-waves, and evolution of fracture with formation of a complex fault zone. Unfortunately, even with the fastest computers available, only the fracture development in a lattice of 100 × 100 elements can now be studied effectively. Since we need to study the 3-D fracture of solids, an opportunity for modelling the structures large enough to display the scaling behaviour will not be available for several more years.

Thus the scaling behaviour of earthquake distributions has to be addressed by other means than the computer simulation of the full continuum-mechanical models. The renormalization technique is the proper tool to rigorously investigate scaling. Recent applications of the renormalization to earthquakes and material failure (Newman and Gabrielov, 1991; Newman et al., 1994) indicate the limits of the computer simulation. For example, the strength of hierarchically arranged fiber bundles with \( n \) elements decreases as \((\log n)^{-1}\); such slow decay would not be apparent in any straightforward computer simulation. Only a very simple model arrangement can now be investigated exactly: the strength and the rupture time distribution for a countable number of fiber elements. To understand earthquake dynamics we need to renormalize the stress tensor fields in a 3-D continuum. The solution of the problem is still far ahead.
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