Seismicity: Turbulence of Solids

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1. Introduction

Since the end of the 19th century it has been known that earthquake occurrence exhibits scale-invariant properties: The frequency of aftershock occurrence decays in time as a power-law and the distribution of earthquake size is also a power-law. Recently, it has been determined that several other statistical features of earthquakes, i.e., spatial distribution of earthquakes and rotation of their focal mechanisms, are also scale-invariant. Seismicity is also recognized as an extremely chaotic phenomenon. Numerous efforts to predict earthquakes, using the history of previous events as well as other available information, have not been successful thus far. With the rapid development of nonlinear mechanics in 1970–1980, several geophysicists, solid state physicists, and applied mathematicians were motivated to explain and simulate these observed phenomena in terms of various nonlinear models. In this paper, I review experimental evidence for earthquake scale-invariance, discuss mechanical and other models proposed to reproduce these properties of seismicity, and, finally, offer the model of random defect interaction which, without additional assumption, seems to explain most of the available empirical results. I argue that there is great similarity between two modes of condensed matter deformation, that of fluid turbulence and that of the brittle fracture of rock material which results in earthquakes. This similarity might be used to gain a better insight into both of these phenomena.

Omori [1895] was the first investigator to note that the after-shock rate decays as the power-law with time. Ishimoto and Iida [1939] and, later, Gutenberg and Richter [1944] found that earthquake size distribution also follows a power-law. Recently, many other earthquake distributions have been found to be scale-invariant or fractal (Takayasu, [1990], p. 31). Kagan [1991a; 1991b] and Kagan and Jackson [1991] list several attempts to

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determine scale-invariant features of earthquakes. At this stage of
our investigation, I believe it is not sufficient to reveal the scale-
invariance of certain earthquake distributions and determine its
fractal exponent (or dimension): Such determination should be
accompanied by analysis of possible errors, as well as by estab-
lishing a possible universality of observed fractal features. Oth-
erwise, the criticism by Kadanoff [986] is often applicable to
such investigations: Due to the lack of organizing theory, it is
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Several models of earthquake rupture and occurrence have been proposed. All of these models greatly oversimplify the real
mechanics of the earth's crust. The standard model of earthquake
source (Burridge and Knopoff, [1967]; Aki and Richards, [1980];
Scholz, [1990]) is a motion of two rigid blocks separated by a
single smooth, usually planar, surface. The resistance of rocks to
the motion is described as a static and dynamic friction. The
Burridge-Knopoff model was a significant first step in quantita-
tive analysis of seismicity: With very simple initial assumptions,
\ it reproduces some obvious features of earthquake occurrence,
\ i.e., "stick-slip" behavior and rupture propagation. Recently,
\ this model and its modifications have been used extensively
(Carlson and Langer, [1989]; Carlson et al., [1991]; Brown,
\ Scholz, and Rundle, [1991]; Huang and Turcotte, [1990]) to
\ explore the size distribution of simulated earthquakes, temporal
\ features of synthetic sequences, chaotic behavior of seismicity,
\ etc. In later sections, I review the size and temporal earthquake
distributions. Here, I want to make some observations on mod-
\ eling of earthquake occurrence as deterministic chaos with a few
degrees of freedom. Because many very simple nonlinear sys-
\ tems exhibit chaotic dynamics, it is not surprising that simple
\ mechanical models of earthquakes yield chaos. However, we
\ have relatively little information on the possible behavior of more
\ realistic nonlinear models with potentially infinite number of de-
\ grees of freedom (cf. Lumley, [1990], p. 258f). However, it is
\ important that even if these simple systems exhibit random, cha-
\ otic behavior of simulated earthquake occurrence, we are much
\ more likely to find chaos and randomness in more realistic, more
\ complex models, and, of course, in natural earthquakes.

There are many other computer models of an earthquake oc-
\ currence; a relatively comprehensive review is given in books by
\ Herrmann and Roux [1990] and Takayasu [1990]. Again, one
\ feels overwhelmed by the diversity and multiformity of the mod-
\ els, of the results of simulations, and by the lack of general
\ physical understanding. Some of the models, such as percolation,
\ neglect to take into account the mechanical properties of the
\ medium. Other models also use unphysical or unrealistic assump-
\ tions, and simplistic geometries of earthquake faults or of me-
\ dium structure. Without a substantial theoretical base, it is
difficult to evaluate the results of these simulations and judge
\ their relation to earthquake fracture (cf. Kadanoff, [1986]).

Recently, a new class of dynamical systems exhibiting self-
\ organized criticality (SOC) has been proposed by P. Bak and
\ others (Bak and Chen, [1991]; Chen, Bak, and Obukhov,
\ [1991]). The SOC models explain the appearance of scale-
\ invariant features in complex dissipative dynamic systems by
\ self-organization of the system into a critical stationary state, a
\ state which is robust with regard to modification of the system.
\ Models based on the SOC idea have been extensively used to
\ reproduce earthquake occurrence (ibid.; Ito and Matsuozaki,
\ [1990]; Takayasu, [1990]; Nakanishi, [1991]). However, most of
\ these models neglect to take into account the long-range, tensor
\ character of elastic stresses which cause earthquakes; thus, the
\ models are not sufficiently specific to simulate earthquake rup-
\ ture and interaction. Thus far, only the distribution of earthquake
\ size has been obtained from these computer simulations; this
\ distribution is the least informative of all scale-invariant features
\ of seismicity (see Section 3).

In our work of the last decade we tried to find statistical rela-
\ tions describing the earthquake process, with the hope of discov-
ering an acceptable physical explanation of widely observed
\ scale-invariant features of seismicity. Thus, it is very important
\ not only to ascertain fractal properties of earthquakes, or their
\ chaotic behavior, but to find the values of power-law exponents or
\ fractal dimensions, to show that these values are universal, and
to try to find a physical model which should explain these phe-
nomena. A kinetic model of earthquake occurrence has been
\ proposed (Kagan and Knopoff, [1981]; Kagan, [1982]). In this
\ model, following a suggestion by Vere-Jones [1976], we apply a
\ critical branching process to simulate earthquake sequences
\ which seem to have all of the known statistical properties of real
\ earthquakes. The scale-invariant features of synthetic sequences
\ are obtained in this model by a trial-and-error procedure. How-
\ ever, since that time, we have shown (Kagan and Knopoff,
\ [1987]; Kagan, [1990]) that the basic assumptions of our model
can be justified by stress tensor interaction caused by random
\ defects. Therefore, it gives us hope that a similar model based on
\ the mechanics of defects might be successful in explaining earth-
\ quake occurrence.

Almost all of the models above use a quasistatic case, i.e.,
inertial effects are disregarded. However, during earthquakes, a
rupture propagates with a velocity which is close to the velocity
of shear waves. Modeling of the dynamic case is an immensely
difficult task, and only for simple systems, such as the Burridge-
\ Knopoff model, is full dynamic consideration possible. We men-
tioned earlier that many simplifications have been made in this
\ model. These simplifications make the interpretation of simula-
\ tion results for this and similar models rather difficult.

Several features of seismicity allow us to compare it with the
turbulence of fluid flow (Monin and Yaglom, [1975]; Frisch and
\ Orszag, [1990]; Lumley, [1990]; Hunt, Phillips, and Williams,
\ [1991]): (a) they both share inherent randomness; (b) their major
\ statistical ingredients are scale-invariant; (c) they both have hi-
\ erarchically organized structures; (d) the size of major structures
\ that control deformation patterns in both cases is comparable to
\ the maximum size of the region; (e) the scale of self-similar
\ structures extends over many orders of magnitude, i.e., for tur-
\ bulence from submillimeter scales to intergalactic distances, for
\ seismicity, again from about a millimeter to thousands of kilo-
meters; (f) both phenomena are intermittent in time and space,
\ although the degree of seismicity intermittency, as reflected in its
\ fractal exponents, is higher than that of fluid turbulence (see
\ below); and, finally, (g) both are complex continuum-mechanical
\ systems with very large (potentially infinite) numbers of de-
\ grees of freedom. Of course, there are many differences between
\ these phenomena as well: (a) the medium of deformation is fluid in
\ the first case and solid in the latter case; (b) in most turbulent flows,
\ the fluid moves with relatively low velocity (significantly lower
\ than the speed of sound), whereas earthquake rupture propagates
\ (over hundreds of kilometers for large earthquakes) with a speed
\ that is close to the shear velocity in a medium; (c) most earth-
\ quake deformation is the effect of dislocations (translational de-
\ fects), whereas disclinations (rotational defects) play a
\ subordinate role (Kröner and Kirchgässner, [1986]; Edelen and
\ Lagoudas, [1988]). In turbulent motion of fluid, vortices (disclina-
\ tions) are primary vehicles of deformation. However, we be-
\ lieve that eventually the comparison of the two modes of condens-
\ ed matter deformation will yield significant new insight into
\ the mechanics of both phenomena.
In what follows, I review the most important properties of seismicity, discuss modeling of seismicity, and again comment on the parallels among the turbulent motion of fluids, turbulence of gravitational bodies, and seismicity patterns, and on the importance of scale-invariance for all of these phenomena (see Mandelbrot, [1983]; Peebles, [1989]; Frisch and Orszag, [1990]). I hope that the results reported herein will contribute to our understanding of earthquake occurrence and its place in a wider picture of turbulent behavior of condensed matter. Actually, I believe that major advances in theoretical understanding of turbulence should greatly benefit all of the latter fields of physics.

2. Earthquakes

As a first approximation, an earthquake may be represented by a sudden shear planar failure—appearance of a large dislocation loop (Aki and Richards, [1980]; Madariaga, [1981]) in rock material. In Figure 1(a) we show a fault-plane trace on a surface of the earth (similar to viewing from above an earthquake occurring on the San Andreas fault). Earthquake rupture starts at the point on the fault-plane called the hypocenter (the epicenter is a projection of the hypocenter on the earth’s surface) and propagates with a velocity close to that of shear waves (3.0–4.5 km/s). As a result of the rupture, two sides of the fault surface are displaced relative to each other in the direction of the arrows; for large earthquakes, the displacement is of the order of a few meters.

Modern plate tectonics, which were formulated in the 1960s, explain earthquake occurrence as a consequence of tectonic plate motion. In ductile mantle material, this deformation is accomplished through a plastic flow of rocks; because the upper crust of the earth is too cold and, therefore, too brittle to flow, it deforms through earthquakes. The rate of plate motion is of the order 1–10 cm/year; thus, over the centuries several meters of deferred displacement may accumulate and then be released mostly in large earthquakes (see Section 3).

The earthquake rupture excites seismic waves which are registered by seismographic stations. In the absence of internal boundaries in the homogeneous and isotropic rock material, two types of waves propagate from the source: primary, compressional (P-waves) and secondary, shear or S-waves. The best modern stations provide digital registration of three components of the ground motion in a very broad frequency range. The seismograms obtained are processed by special computer programs to deduce properties of earthquakes. Preliminary seismogram inversion is accomplished about an hour after an earthquake occurrence.

The routine results of seismogram inversions characterize earthquakes by origin time, hypocenter position, and by the second-rank symmetric seismic moment tensor for each event. In the system of coordinates shown in Figure 1(b), the tensor matrix is

\[
M = \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 
\end{pmatrix}
\]

where \(M\) is a scalar seismic moment of an earthquake. In an arbitrary system of coordinates, all entries in (1) might be nonzero. However, the tensor is always traceless, with a zero determinant; hence, it has only four degrees of freedom—one for the norm of the tensor (scalar seismic moment) and three for orientation (they define the focal mechanism of an earthquake). Another equivalent representation of the earthquake focus is a quadrupole source of a particular type, known in seismology as a double-couple (Aki and Richards, [1980]).

Figures 1(a) and 1(c) show two graphic representations of an earthquake source: double-couple configuration of equivalent forces and quadrupolar radiation patterns characteristic for earthquakes. The focal plots that are standard in seismology involve painting on a sphere the sense of the first motion of P-waves, solid for compressional motion and open for dilatational. Two orthogonal planes separating these areas are the fault and auxiliary planes. In routine determination of focal mechanisms, it is impossible to distinguish between these planes. The intersection of these planes is the null-axis (N-axis), the P-axis is in the middle of the open lune, and the T-axis is in the middle of the closed lune. These three axes are called principal axes of an earthquake focal mechanism, and their orientation defines the mechanism.

Magnitude \((M)\) is an empirical measure of the earthquake size (approximately proportional to the logarithm of scalar seismic moment). Another measure of earthquake size, the released seismic energy is not evaluated directly and is a scalar; thus, the seismic moment tensor is a preferred quantity for earthquake characterization. Data prior to the introduction of worldwide earthquake digital recording (i.e., before 1977) were usually in more restricted form: earthquake focal mechanism had not been determined, at least not routinely for most earthquakes, and magnitude was used for characterization of earthquake size.

Earthquakes are usually subdivided by the depth of their focus into shallow (depth 0–70 km), intermediate (70–280 km), and deep (280–700 km) events. Shallow events exhibit strong clus-

![Figure 1](image-url)
tering in time; large earthquakes are usually followed by se-
quences of smaller earthquakes, called aftershocks; strong
aftershocks, in their turn, often have their own subsequences of
events, etc. Occasionally, large earthquakes are preceded by
foreshock sequences consisting of one or several weaker events.
The large event that dominates the sequence is called the main-
shock.

The earthquake data are assembled in catalogs, of which the
most complete and homogeneous at present is the catalog of
moment tensor inversions compiled by the Harvard group
(Dziewonski et al., [1991]). The available catalog covers the
period from January 1, 1977 to May 31, 1991, and contains close
to 10,000 events. In principle, if we know these earthquake pa-
rameters, we can calculate the low-frequency motion they excite,
and, henceforth, the deformation history of the earth’s brittle
crust. Therefore, we have complete information about the seismic
process.

Even casual inspection of earthquake catalogs reveals the ran-
dom nature of earthquake occurrence, thus making it imperative
to use statistical methods for their interpretation. As an example
of earthquake data, in Figure 2 we display focal mechanisms for
earthquakes in the Los Angeles area. It is obvious, even from this
diagram, that earthquakes are not concentrated on a few faults
and the mechanisms of neighboring events may have very dif-
ferent orientations, i.e., they undergo large three-dimensional
(3-D) rotations.

There are many other methods of earthquake study. Detailed
analysis of seismograms reveals, for example, a complex internal
temporal and spatial structure of earthquake sources. Geological
investigations of earthquake fault traces similarly disclose their
complicated geometry. Geodetic observations allow us to study
pre- and postseismic deformation, caused by earthquakes, etc.
Unfortunately, these data usually lack uniform coverage of all
events, which makes them less suitable for statistical analysis.

In our work of the last decade, we analyze statistically several
catalogs of tectonic earthquakes to study the interrelations be-
tween earthquakes and to further theoretical understanding of
the earthquake process. This form of information in earthquake cat-
logs suggests the type of models which should be used to de-
scribe seismicity—stochastic, multidimensional, tensor-valued,
point process: \( M \times T \times \mathbb{R}^3 \times SO(3) \), where \( T \) is time, \( \mathbb{R}^3 \) is the
3-D Euclidean space, and \( SO(3) \) is the group of 3-D rotations.
Two major statistical techniques are used in the analysis: the
maximum-likelihood procedure and the moment-function
method. Results of this statistical analysis are reported in Sec-
tions 3 to 6.

3. Size Distribution—M

The distribution of earthquake sizes is usually invoked as a first
confirmation for virtually any model of seismicity. As mentioned
earlier, the magnitude distribution follows the Gutenberg–Richter
(G–R) relation. This relation can be transformed into the power-
law (Pareto) distribution for the scalar seismic moment with the
density \( \phi(M) \sim M^{-\beta} \). Statistical analysis of magnitude and
seismic moment distributions yields the value of \( \beta \) between 0.5
and 1.0 for small and medium-size earthquakes. Simple consid-
erations of finiteness of the seismic moment or deformational
energy, available for an earthquake generation, require that the
power-law relation be modified at the maximum-size end of the
moment scale (Kagan, [1991b]). At the minimum, the distribution
density tail must have a decay stronger than \( M^{-\beta-1} \) with
\( \beta > 1 \). Usually this problem is solved by the introduction of an
additional parameter, called a “maximum magnitude” (\( M_{\text{max}} \)), to
the distribution. This new parameterization of the modified G–R
relation has several different forms (Kagan, [1991b; 1992b]). All
of these distributions predict that the major part (about 50% or
more) of the total seismic moment is released by very large
earthquakes, i.e., events with \( (M_{\text{max}}/10) < M < M_{\text{max}} \).

Among modifications proposed to alter the distribution at its
high end, there is the “characteristic” earthquake hypothesis
(Scholz, [1990]; Brown et al., [1991]) which states that although
the size distribution of earthquakes for a large area follows the
power-law, for individual faults or fault segments, it has a sig-
nificantly different form: The largest earthquakes form a sepa-
rate, independent population with the frequency about an order
of magnitude higher than the frequency obtained by extrapolation
of the curve from small events. This hypothesis has an added sig-
nificance since several attempts have been made to obtain a sim-
ilar distribution using the Burridge–Knopoff [1967] model of an
earthquake fault (Carlson and Langer, [1989]; Brown et al.,
[1991]; Carlson et al., [1991]; see also Levi, [1990]).

It can be shown (Kagan, [1992b], and references therein) that
the statistical proofs of this hypothesis which have been published
so far have serious defects, and therefore fail to validate the
characteristic earthquake hypothesis. The observed pattern of
earthquake size distribution can easily be explained by a simplier
assumption of the Pareto distribution truncated at the \( M_{\text{max}} \) or the
gamma distribution (see below). The appearance of two different
populations of events in theoretical modeling of earthquake rup-
ture propagation (Carlson and Langer, [1989]; Carlson et al.,
[1991]; Brown et al., [1991]) is caused, most probably, by the
presence of two sets of springs in the Burridge–Knopoff model of
earthquake fault. Because these springs have different stiffnesses,
we should expect this model to yield two populations of slip
events. However, rocks in the earth’s interior are known to have
the same Poisson ratio (of the order of 0.25); hence, their elas-
ticity constants (like Lamé’s constants) should depend on only
one parameter. This implies that the emergence of two different earthquake populations in some modifications of the Burridge–Knopoff model might be due to the properties of the model, not to the properties of the earth.

In Figure 3 we display cumulative histograms obtained from the Harvard catalog of seismic moment solutions for three types of earthquakes: shallow, intermediate, and deep. The curves are flat at small values of $M$, reflecting lack of these events in the catalog due to the finite detection threshold of the seismographic network. This flat part of the curve is followed by a scale-invariant part. At large $M$, the curves are bent downward, the lack of very strong earthquakes being due to the above-mentioned finiteness of the seismic moment flux.

The brittle crust layer is relatively thin (about 10–20 km); thus, one should expect that the size distribution exhibits scaling breakdown (Carlson et al., [1991], eq. A3) around $M \approx 10^{18}$–$10^{20}$ Nm (linear size of these earthquakes corresponds to the thickness of the layer). But the curves display no such crossover. A usual explanation for this fact is that in the worldwide seismicity we see a mixture of distributions belonging to many different tectonic regions; thus, the difference is smoothed. However, there must be a global mechanism which yields an almost perfect power-law relation in these moment limits, and apparently the mechanism is impervious to the change of dimensionality of earthquake faulting. For small earthquakes (with the size of a focal region less than 15 km), the rupture propagates along a 2-D surface in a 3-D space, whereas for large shallow earthquakes, the rupture is constrained in a horizontal direction and, therefore, is concentrated essentially in two points [see Fig. 1(a)]; hence, it is 1-D in the 2-D space. However, as seen from Figure 3, the distribution for shallow seismicity does not display any significant trace of that crossover. Similarly, intermediate and deep earthquakes which occur in a crustal slab subducting into the mantle (Madariaga, [1981]) do not show a significant self-similarity breakdown around $M = 10^{18}$ Nm.

We assume that the seismic moment is distributed according to the gamma distribution (Kagan, [1991b]) with the probability density

$$\phi(M) = C^{-1} M^{-1-\beta} \exp(-M/M_{\text{max}}),$$  \hspace{1cm} (2)

where $C$ is a normalizing coefficient. Among the two-parameter distributions, this law has an advantage of being the simplest model consistent with the data and other available information. By the simplest model, I mean a distribution having the maximum entropy, i.e., a model, which under known constraints (a distribution which is a power-law for small and medium-size earthquakes and has a finite first moment), maximizes the uncertainty of our knowledge (Kagan, [1991b]).

The maximum likelihood procedure allows us to retrieve the values of parameters for all three earthquake categories (to ensure the data uniformity, we use events with $M \geq 10^{18}$ Nm): shallow events have $\beta = 0.63$–0.73, $M_{\text{max}} = 1.1 \times 10^{21}$–6.3 $\times 10^{22}$ Nm; intermediate events have $\beta = 0.37$–0.65, $M_{\text{max}} = 1.4 \times 10^{22}$–2.8 $\times 10^{23}$ Nm; and deep events have $\beta = 0.25$–0.75, $M_{\text{max}} = 4.8 \times 10^{20}$–7.9 $\times 10^{21}$ Nm. The above limits correspond to the 95% confidence area. (In the gamma distribution, some earthquakes may have the moment $M > M_{\text{max}}$.) The value of $M_{\text{max}}$ for shallow events implies that the size scale-invariance breaks down for earthquakes with $m = 8.6 \pm 0.25$, i.e., for the linear focal size of 100–300 km (Madariaga, [1981]). Shallow earthquakes differ from other events by the presence of a large number of aftershocks (catalogs of shallow earthquakes contain up to 30%–40% of aftershocks; Kagan, [1991b]). If we take these aftershocks into account and consider the size distribution of earthquake sequences, the value of $\beta$ for shallow seismicity will be comparable to that of deeper events, i.e., $\beta = 0.5$ (ibid.).

The earthquake size distribution is the least informative of all the power-laws that govern an earthquake occurrence: percolation models with the lattice dimension $d \geq 6$; self-organized criticality models with $d \geq 4$; a critical branching process—all yield a power-law cluster size distribution with $\beta = 0.5$ (Kagan, [1991b]). Thus, the power-law size distribution can be obtained from a variety of models, and the fact that some events in a model follow a fractal distribution, even with a correct exponent value, does not prove that the model can be used to describe seismicity.

In our modeling of earthquake ruptures through a critical branching process (Kagan, [1982]), we use a branching random walk in six dimensions: three dimensions of the Euclidean space and three dimensions of the rotation group SO(3). In this model, translations and rotations of fractures both contribute to complex deformation of an elastic solid. If this hypothesis is true, then we see no crossover for earthquakes with focal size of about 15 km (see above) because the total dimension of the phase space changes from 6 to 5. Thus, the value 0.5 for $\beta$ may be a universal constant.

4. Time Distribution—$T$

Orois [1895] and several other researchers have shown that the frequency of aftershock occurrence decays in time as $T^{-p}$, where the exponent $p$ is slightly larger than 1.0 (Kagan and Jackson, [1991]; Kagan, [1991c]). Foreshocks have been found to follow a similar pattern, their number increasing as the mainshock approaches, as a power-law with a similar value for the exponent. Similar scale-invariant distribution of aftershocks is found for acoustic emission during microfracturing of rock specimens (Hirata, [1987]).

![Figure 3](image-url)
There are several problems involved in a statistical study of temporal earthquake patterns: one is the instability of statistical estimates for a decay exponent due to general difficulties of statistical analysis of power-law (or stable) distributions (Zolotarev, [1986]; Kagan, [1991c]). Moreover, straightforward investigations of individual aftershock sequences are less reliable because samples are biased by selection (only large aftershock sequences can be studied, whereas even many strong earthquakes do not produce such sequences) and by secondary clustering of aftershocks. Attempts to determine the fractal dimension of temporal projection of earthquake sequences have not been completely successful (Kagan and Jackson, [1991]) because the dimension depends on the degree of spatial averaging.

Therefore, we attempt to study the temporal behavior of earthquakes by simulating the sequences, processing them through the same maximum likelihood procedure that is applied to natural earthquakes, and then comparing both types of results (Kagan and Knopoff, [1981]; Kagan, [1991c]). In the simulations, we allow elementary seismic events to develop according to a critical branching process; the probability for each event to produce another event in a time interval \( dt \) is proportional to \( t^{-3+\beta} dt \). The graphs of cumulative seismic moment produced by this model clearly show patterns of clustering of different orders. The source–time function appears to consist of several steps, each starting with a sharp onset and usually becoming more gradual later. Thus, the sequence has a few foreshocks and many aftershocks, as do natural earthquake sequences. By its construction, a simulated earthquake sequence is an asymmetric Cantor set.

Processing the simulated earthquake sequences by the maximum likelihood procedure, we find that the value of \( \beta = 0.5 \) for \( \theta > 0 \) produces sequences with few, if any, aftershocks; the almost total absence of aftershocks is the property of intermediate and deep earthquakes. All earthquakes have a power-law size distribution with the value of \( \theta = 0.5 \) (Kagan and Knopoff, [1981]).

The power-law temporal dependence may have a simple explanation: If we assume that stresses at the end of an earthquake rupture are below the critical value and thereafter change randomly according to a one-dimensional Brownian motion, then the level-set of this motion is a fractal set with dimension 0.5 (Man- delbrot, [1983]). Therefore, the stress again reaches the critical level at time moments distributed according to a power-law (Kag- an and Knopoff, [1987]). Short-term clustering of earthquakes is then due to stress diffusion.

Attempts to reproduce Omori’s law of aftershock rate decay by computer simulations are usually based either on a special mechanism for temporal scale-invariance (Ito and Matsuzaki, [1990]) or on the introduction of time delay into a mechanical model. It is worth mentioning that almost all simulations of earthquake sequences using the Burridge–Knopoff model or other mechanical models (Carlson and Langer, [1989]; Carlson et al., [1991]; Brown et al., [1991]; Nakamishi, [1991]; Herrmann and Roux, [1990]; Huang and Turcotte, [1990]; Chen et al., [1991]) fail to reproduce this most obvious feature of an earthquake occurrence.

It is more difficult to study long-term properties of an earthquake occurrence: available catalogs are usually too short compared with recurrence times of the largest events (decades and centuries). There have been continuing attempts in seismology to prove that earthquakes, especially very large ones, are periodic or quasiperiodic. Nishenko and Buland [1987] analyzed about 50 pairs of earthquakes that occurred during the last several hundred years, postulating that the interearthquake time is distributed according to the log-normal distribution with a relatively small coefficient of variation (0.21). Their study seems to be confirmed by the simulation of earthquake sequences using the Burridge–Knopoff model (Carlson and Langer, [1989]; Huang and Turcotte, [1990]; Brown et al., [1991]): synthetic sequences of the strongest events are also quasiperiodic in these investigations. However, claims of earthquake quasiperiodicity should be treated with great caution: If we calculate the total number of events available for analysis by Nishenko and Buland [1987]—magnitude 6 and larger events which occurred during last few hundred years—the number is of the order of tens of thousands (see Fig. 3). Of course, not all earthquakes have been recorded in historical, instrumental, and other catalogs, but available records, nevertheless, contain thousands of such events. A possibility for biased sampling of the phenomenon which exhibits a great degree of randomness is very strong; thus, it is very important that the events selected for statistical study should be representative of the general behavior of earthquakes. Kagan and Jackson [1991] analyze all earthquakes available in the instrumental catalogs: the coefficient of variation for interearthquake times is consistently higher than 1.0; hence, we conclude that the most important feature of seismicity is long-term clustering.

The fundamental condition for earthquake periodicity is the degree to which the deformation energy (or stress) is depleted by the strongest earthquakes. If a significant part of stored energy is released during the largest earthquakes, it seems feasible that a “dead” period should follow. Evidence for the state of postseismic stress from field observations is inconclusive. Kagan and Jackson [1991] suggest that even after strong earthquakes which rupture the whole earth crust, the numbers of equally strong events decay in time according to a power-law, and thus contrary to the expectations of the “periodic” hypothesis. These results are consistent with an idea that tectonic stresses are always high and are close to the critical value even after a large earthquake. In the Burridge–Knopoff model, a large slip event releases most of the elastic energy, which in this model is stored only in the springs. Thus, a new large event is possible only after a significant period of time has elapsed. In the earth, the potential elastic energy may be stored in tectonic plates or in the mantle which measure thousands of kilometers; therefore, even the largest earthquakes may release only a small part of the energy. Moreover, a fracture depends, in principle, on three stress tensor invariants (Kagan, [1990]); stress variations caused by large earthquakes may modify an occurrence of subsequent events in a very complex manner. Initiation and propagation of earthquakes should also depend on 3-D topological properties of stress singularities induced by the past deformation history (cf. Gabriely and Keilis-Borok, [1983]).

To study the long-term properties of seismicity, we statistically analyze (Kagan and Jackson, [1991]) several instrumental earthquake catalogs. After we take into account the effect of short-term clustering (aftershocks), the degree of clustering in residual catalogs is the same for earthquakes in different depth ranges. Therefore, we conclude that time clustering of earthquakes is a universal phenomenon which has two general features: (1) a short-term, strong clustering of shallow earthquakes responsible for foreshock–mainshock–aftershock sequences and (2) long-term, weak clustering which characterizes all mainshock earthquakes—shallow, intermediate, and deep. There is circumstantial evidence that long-term variations of seismicity as well as short-term clustering are governed by a power-law temporal distribution, i.e., they are fractal. The fractal dimension of the set of earthquakes on the time axis is of the order of 0.8–0.9; main- shock occurrence is much closer to a stationary Poisson process than standard aftershock sequences of shallow earthquakes. We conclude that both short- and long-term earthquake behavior can be explained by fractal distributions with the value of the expo-
tent \( \theta = 0.5 \) for shallow events and \( \theta = 0.8-0.9 \) for deep earthquakes.

A tentative interpretation of our results is as follows. Because long-term clustering is a property of earthquakes belonging to all depth ranges, the clustering is due to dynamic processes in the earth's mantle. To anticipate that value of fractal dimension \( \theta \) equal to 0.5 corresponds to temporal clustering of events during a fracture of brittle solid materials. This value of \( \theta \) yields a highly discontinuous release of seismic energy (Kagan and Knopoff, [1981]), usually interpreted as fore-, main-, and aftershock sequences. The value \( \theta \) between 0.5 and 1.0, on the other hand, should correspond to a plastic flow of materials and instabilities accompanying that flow.

These conclusions can also apply to the deformation of solids and fluids. An ideal solid body (without defects) should fail in one episode of fracture (\( \theta = 0 \)); however, all natural rocks have, of course, many defects. Hence, aftershocks and foreshocks appear which increase the dimension to 0.5. For fluids, the usual reasoning is reversed. Even for plastic material with very high viscosity, standard models imply a continuous deformation; hence the dimension is 1.0. Therefore, one should expect that nonideal, viscous fluids should have deformation with episodic (fractal), temporally intermittent features which would result in lowering the dimension.

5. Spatial Pattern—\( \mathbb{R}^3 \)

The fractal spatial structure of rock fracture has attracted many investigators. Fractal sets can readily be visualized in this case; hence, the intuitive appeal of the measurements is obvious. These studies confirm spatial self-similarity of fracture and produce several values for the spatial fractal dimension, \( \delta \), of earthquake fracture (Kagan, [1991a]). However, I argue that many of these measurements do not relate directly to the earthquake fracture because they measure the end product of the fracture process: fault traces or fracture rock fragments. These objects can be strongly influenced by decompression (absence of lithostatic pressure) and by the influence of a free boundary (ibid.), such as the earth’s surface. Even the measurements of the fractal dimension of a set of earthquake hypocenters are subject to various types of biases and errors; among the most important are a time span of a catalog, hypocenter location errors, and projection of hypocenters on the earth’s surface. Failure to take these errors into account would strongly influence the value of the fractal dimension (ibid.).

Using several catalogs of earthquakes, we analyze the distribution of distances between pairs of earthquake hypocenters to determine the spatial fractal dimension of an earthquake fracture. As the time span of the catalog increases, \( \delta \) asymptotically reaches the value 2.1–2.2 for shallow earthquakes. Approximatively the same asymptotic value of dimension is obtained for a catalog of earthquakes with aftershocks removed. The value of fractal dimension declines to 1.8–1.9 for intermediate events and to 1.5–1.6 for deep events. In comparison of the above values, one needs to take into account that most of the relative motion between tectonic plates is accounted for by brittle shear deformation caused by shallow earthquakes, whereas for intermediate and deep seismic events, most of the rock deformation is aseismic. As a conjecture, I propose that the value of \( \delta \) should be less dense for solids than for fluids.

Taking into account various possible errors and biases, we conclude (Kagan, [1991a]) that the fractal dimension of brittle shear fracture of rocks is \( 2.20 \pm 0.05 \). This dimension is smaller for intermediate and deep earthquakes because they represent only a small part of rock deformation at great depths. The results appear to rule out the conventional model of earthquake hypocenters occurring on a single isolated plane or on several planes and require instead that a fault zone for shallow earthquakes be nonplanar and fractal.

6. Rotation—\( \text{SO}(3) \)

Kagan [1982] introduced the rotational Cauchy distribution (see below) to represent rotations of focal mechanisms of microdislocations which comprise the focal zone of an earthquake. The Cauchy distribution is especially important for representation of
earthquake geometry because it can be shown by theoretical arguments (Zolotarev, [1986], pp. 45-46; Kagan, [1990]) and by simulations (Kagan, [1990]) that the stress tensor in the medium with defects follows this distribution. It is difficult to measure the stress tensor itself in the deep interior of the earth, but we may infer the stress pattern on the basis of stress singularities, the sudden onset of which are registered as earthquakes. In particular, rotations of earthquake focal mechanisms give us an indication of the stress redistribution. We argue (ibid.) that the Cauchy distribution of the stress should produce the rotational Cauchy distribution earthquake sources.

The Cauchy law is a stable distribution (Mandelbrot, [1983]; Zolotarev, [1986]; Takayasu, [1990]). The stable distributions are important to us for two reasons: (a) They are invariant under addition of random variables—suppose that two independent stochastic variables \( X_1 \) and \( X_2 \) are distributed according to the one-parameter Cauchy distribution with the parameter values \( \kappa_1 \) and \( \kappa_2 \). The distribution of their sum, \( X \), is a convolution of the two distributions

\[
F_X(x) = F_{X_1}(x) * F_{X_2}(x),
\]

where \( \kappa = \kappa_1 + \kappa_2 \). This invariance under addition is the reason the distributions are called stable (Mandelbrot, [1983], pp. 367f). (b) The stable distributions have a power-law tail, i.e., they are scale-invariant.

As a result of the 3-D rotations of elementary dislocations comprising the earthquake source zone, the resulting complex extended source should contain rotational dislocations (disclinations) which we identify with asperities/barriers controlling the initiation, propagation, and stopping of earthquake fractures. The general rotational Cauchy distribution can be written as (Kagan, [1982; 1990])

\[
F(\Phi) = \frac{2}{\pi} \left[ \arctan (\frac{\tan(\Phi/2)}{\kappa}) - \frac{\tan(\Phi/2)}{\kappa} \right],
\]

where \( A = \tan(\Phi/2) \) and \( \Phi \) is the rotation angle. The parameter \( \kappa \) of the Cauchy distribution represents the degree of incoherence or complexity of an earthquake fault.

An additional complication in the study of the 3-D rotation of earthquake focal mechanisms is the symmetry of the source: The double-couple earthquake source has a rotational symmetry of a rectangular box with unequal sides (the dihedral group \( D_3 \)). Due to this symmetry, the maximum rotation angle for the earthquake source cannot exceed 120° (Kagan, [1990]).

Using the correspondence between the group \( SO(3) \) and the group of normalized quaternions, we have solved an inverse problem of a 3-D rotation of double-couple earthquake sources; that is, for each pair of focal mechanisms we find a 3-D rotation which rotates one mechanism into another (Kagan, [1992a]). In Figure 4(a), we display histograms for the distribution of rotation angle \( \Phi \) for shallow earthquake pairs which are separated by a distance of less than 50 km. We study whether the rotation of focal mechanisms depends on where the second earthquake of the pair is located with regard to the first event. Thus, we measure the rotation angle for hypocenters located in 30° cones around each principal axis (curves, marked the T-, P-, and N-axes) of the first event (see Figs. 1 and 2). The curves in Figure 4(a) are narrowly, clustered and are obviously well-approximated by the rotational Cauchy distribution.

Figure 4(b) shows \( \Phi \) histograms for shallow earthquakes separated by 400–500 km. Here the curves correspond to fault-planes (the N-axis) are clearly separated from the histograms connected with the T- and P-axes; whereas the rotation near the fault-plane is relatively small (\( \kappa = 0.2 \), the earthquakes which are situated in cones around the T- and P-axes have focal mechanisms which are essentially uncorrelated with the primary event.

The curves are close to the curve corresponding to completely random rotation of a double-couple (Kagan, [1990; 1992a]). We obtain similar results for intermediate and deep earthquakes.

The results presented above, suggest that rotations of focal mechanisms follow the rotational Cauchy distribution; by implication, tectonic stresses, which cause earthquakes, are distributed according to the Cauchy distribution. This conclusion is a necessary consequence of the presence of defects in rock material (see above). Because the Cauchy distribution is stable with a power-law tail, its control of stress means that the earthquake rupture in the presence of defects has to produce the fractal geometry.

Kagan [1982] found that, to explain the complex geometry of the San Andreas fault, the value of \( \kappa \) for the rotational Cauchy distribution should be of the order \( 10^{-3} \). This value corresponds roughly to the ratio of the average slip to the length of rupture.

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**Figure 4.** Distributions of rotation angles for pairs of focal mechanisms of shallow earthquakes from the Harvard catalog; hypocenters are separated by distances (a) 0–50 km; (b) 400–500 km.

- Circle—hypocenters in 30° cones around the T-axis;
- plus—hypocenters in 30° cones around the P-axis;
- star—hypocenters in 30° cones around the N-axis. Solid line is for the Cauchy rotation with (a) \( \kappa = 0.1 \) and (b) \( \kappa = 0.2 \); dashed line is for the random rotation.
during an earthquake (Scholz, [1990], pp. 183–186). The small value of \( r \) means that the rupture during an earthquake propagates along almost a single plane [as shown in Fig. 1(a)]; only on a very rare occasion does the rupture branch or deviate significantly from a planar surface. In the above studies, small or infinitesimal deformations (Kanamark material) was investigated. However, the formation of a mature earthquake fault system over time periods of millions of years should involve large deformation with total displacements which are comparable to the size of the region. The earthquake fault system should stabilize over such time periods into a statistically stationary, but evolving, self-organized, geometrically complex pattern.

For earthquakes occurring on a complex fault system, we find the rotations of focal mechanisms which are approximated by the Cauchy distribution with the value of \( \kappa = 0.05\text{–}0.1 \) (Fig. 4). Thus, we might hypothesize that an earthquake fault system starts with a relatively simple fault which exhibits only a slight complexity, like branching, bending, and rotation. For such a fault, the value of \( \kappa \) should be small, of the order \( 10^{-2} \). In the course of its tectonic history, the complexity of a fault system increases; in terms of \( \kappa \) values, this is reflected in a gradual accumulation of rotations [see Eq. (3)]. Therefore, we estimate that to reach \( \kappa = 0.1 \), the fault system should require about \( 10^3 \) to \( 10^4 \) large earthquakes to occur in a fault zone. If the return time for large earthquakes in a fault zone is of an order of a few hundred years, the age of the zone would reach several millions or tens of millions of years, the value which corresponds to standard views of plate tectonics.

Is the fractal dimension of a set of earthquake hypocenters connected in any way with our rotation results? An ideal solid crystal (without defects) should fail along a planar dislocation. Hence, the dimension of fracture should be 2.0; the Cauchy distribution (4) has \( \kappa = 0 \) for such a crystal. However, the results of our measurements of the focal mechanism rotation indicate that, due to the tectonic evolution, focal zones of earthquakes contain partially incoherent defects distributed according to the Cauchy distribution with \( \kappa \) as large as \( 0.2 \). It would be of great interest to know whether \( \kappa = 0.2 \) implies the hypocentral fractal dimension \( d \) of 2.2 (see Section 5).

7. Discussion

In the previous sections, I presented evidence that most empirical earthquake distributions are scale-invariant: the size, time, space, and orientation distributions have a power-law form. I emphasize again that these data characterize an earthquake process completely: If we know the size, time, space, and orientation of earthquakes, we can calculate all the brittle deformation of the earth’s crust. However, we treat the distributions as independent, i.e., in our statistical analysis, we assume that, for example, the size distribution is not dependent on the orientation, and so on. Similar to the study of statistical aspects of the fluid turbulence (Monin and Yaglom, [1975]), more fundamental investigations should aim at the characterization of joint distributions. This is a difficult task both due to lack of extensive earthquake data and computational complexity of the problem.

However, as suggested in earlier sections, the obtained empirical results yield important information. The observed self-similar patterns of an earthquake occurrence are shown to be derived from simple assumptions of the presence of defects in the rock material. In the time domain, Omori’s law of foreshock/aftershock occurrence and, in general, the time clustering of earthquake events has been shown to be a consequence of Brownian-like motion behavior of random stresses due to defect deformation of rock material (Kagan, [1987]). This implies that the fractal dimension of a temporal occurrence of earthquake sequences is equal to 0.5. Similarly, the presence, evolution, and aggregation of defects in the rock medium are responsible for fractal spatial patterns of earthquake faults and rotation of earthquake focal mechanisms (Kagan, [1990; 1992]). A stable Cauchy distribution governs stresses caused by these defects.

The above-mentioned scale-invariant distributions describe earthquakes which have already occurred; that is, we do not have sufficient complete statistical information on the rupture process itself. As mentioned in Section 1, the dynamic modeling of the rupture in all its complexity represents a formidable problem. However, the available evidence indicates that the observed earthquake behavior can be extrapolated toward smaller time and distance intervals. Therefore, we expect that the dynamic modeling of rupture in complex materials with defects will yield scale-invariant features. Indeed, the linear (and nonlinear) elasticity equations do not have any characteristic temporal or spatial scale (however, they do have characteristic velocities: \( V_L \) and \( V_S \), velocities of the longitudinal and shear waves). Thus, the distribution of physical quantities should be scale-invariant.

The above considerations mean that if we know the geometry of defects in a medium, future deformation patterns can be predicted. However, it will never be possible to obtain a complete description of stress singularities. At a certain small-scale level, we need to apply a stochastic representation of the internal geometry of matter. Due to the scale-invariant nature of defect interaction, this representation should use a renormalization technique. The renormalization of a stress pattern that is a tensor-valued field represents a formidable problem. I also hypothesize that the scale-invariant topology of the defects must be effectively described to achieve a more complete understanding of the seismicity. It is of interest to note that fractal distributions in two other important problems in physics, turbulence in fluids and distribution of gravitational forces, can also be shown to be dependent on the presence of defects—vortex tubes in the former case (Takayasu, [1990], p. 99) and gravitational bodies in the latter case (Zolotarev, [1986]; see also Chuang et al., [1991]).

The results reported above make us question the suitability of some notions and models commonly used in the theory of an earthquake source. In particular, the standard models of the source (see Section 1) are based on the mechanics of man-made objects. Three clearly defined geometrical scales can be distinguished in such objects: (a) exterior or macroscopic, which applies to the whole body; (b) interior or microscopic, which describes defects and intergranular boundaries of the object material; and (c) unavoidable irregularities of the surface of the object. The factors connected with the latter scale are usually taken into account by the introduction of frictional forces (Scholz, [1990]).

In many engineering applications, great efforts are expended to keep the friction under control and reduce the surface roughness. Despite these measures, surface irregularities of sliding objects always increase, making the very existence of the separate scales (a) and (c) an unstable situation. Our study of the spatial statistical properties of an earthquake occurrence indicates that there are no separate scales for this process; the process is self-similar and, as such, lacks any intrinsic scale. Scholz ([1990], pp. 50–51) recognizes that the faults’ scale-invariant character differentiates them not only from theoretical planar faults but also from laboratory fault models which have a clear scaling cutoff of their surfaces. Moreover, fault geometry analysis (Kagan, [1982; 1991a]) indicates that earthquakes do not occur on a single (possibly wrinkled or even fractal) surface, but on a fractal structure of many closely correlated faults. The total number of these infinitesimal faults might be infinite. Similarly, scales (a) and (b) are usually separated in materials science and engineering applications by introducing effective properties of the material. These properties can be either mea-
In earthquake studies, we consider the propagation of a rupture through rock material which during millions of years of its tectonic history has been subjected to repeated earthquake deformation. As a result of this process, the largest defects which we identify with the fault systems have approximately the same size as tectonic blocks; hence, no scale separation is possible. These defects might be critically self-organized coherent aggregates of many small earthquake sources (Bak and Chen, [1991]; Chen et al., [1991]). In what follows, I call such materials tectonically self-organized rocks.

On the basis of the preceding discussion, I propose a new framework for the geometry and mechanics of earthquake fault zones. Theoretical strength of materials is two to three orders of magnitude larger than their observed strength (Lardner, [1974], pp. 6–15; Scholz, [1990], pp. 2–3). The difference is thought to be due to defects such as dislocations, inclusions, grain interfaces, etc. The scale of the defects is usually much smaller than the size of an object to be tested. For the purpose of the following discussion we call the former materials "ideal" and latter materials "regular." The shear strength of ideal materials is of the order $\mu$ to $\mu/10$, where $\mu$ is the shear modulus; for regular materials, the strength is $\mu/100$ to $\mu/1000$ (Lardner, [1974]).

An apparent strength of tectonically self-organized materials depends to a large degree on the orientation of the rupture and might again be diminished by factors of about 100 or 1000 compared to regular materials (the effective strength is $\mu \times 10^{-7}$ to $\mu \times 10^{-5}$). In a certain sense, such rock material, deformed under the tectonic stress, has a zero strength. If we define the strength of material by the onset of the nonlinear response, tectonic blocks always have some small earthquakes occurring in them; hence, they always have subvolumes where the stress is at its critical value. Gabriélov and Newman [1990] and Newman and Gabriélov [1991] show, using the renormalization method, that the strength of hierarchically arranged fiber bundles decreases as $1/(\log \log L)$, where $L$ is the size of the system; thus, the strength vanishes asymptotically as $L \to \infty$. We hypothesize that the strength of rocks is essentially the same everywhere. If some blocks of material show little internal deformation, it is not due to their greater intrinsic strength, but, conversely, due to an absence of large defects in these blocks; hence, the strength is a consequence of the previous tectonic history of these blocks (cf. Davy, Sornette, and Sornette, [1990]). Thus, we propose that earthquakes occur on faults not because these faults are weak or have inferior strength, but because previous earthquakes have left very strong internal stresses in their wake, so that relatively small stress increments are needed to fracture the area again.

To facilitate the comparison, I summarize the properties of different types of material in Table 1. As explained in the previous sections, some of the entries in Table 1 are hypothetical, some are based on empirical measurements, and some are speculative but have considerable support from experimental data or simulations. The first entry of the table is, of course, a conjecture. Although the statistical measurements of fracture in specimens of regular materials indicate that the deformation has scale-invariant properties (Hirata, [1987]; Hirata et al., [1987]; Scholz, [1990]; Davy et al., [1990]), an interpretation of these results is fraught with many difficulties, i.e., nonstationarity of fracture in laboratory experiments, multiple-wave scattering from the specimen boundaries, an influence of these boundaries on the stress pattern, dynamic interaction between a specimen and testing equipment, etc. Due to these and other factors, laboratory measurements at present do not determine the seismic moment tensor of microearthquakes, making the experimental data incomplete. Finally, in the last two entries of the table, fractal dimensions for two other modes of stochastic, scale-invariant motion of matter are shown for comparison.

In conclusion, I would like to comment briefly on the possibilities of extending the reported results to future investigations. I envision four possible directions for study of fracture and deformation of solids: (1) mathematical investigations based on gauge theories of defects (Kröner and Kirchgässner, [1986]; Edelen and Lagoudas, [1988]); (2) computer modeling of fracture; (3) laboratory experiments; (4) statistical analysis of earthquake occurrence and its modeling. I believe that to be effective, the mathematical modeling should include the scale-invariance of defects in a continuum-mechanical description of elastic deformation. The stochastic nature of defect geometry also needs to be addressed, so that the theoretical conclusions might be compared with results obtained from a statistical analysis of experimental data. In computer simulations of fracture, we should move toward a full three-dimensionality and complexity of defect geometry with proper consideration for the tensor character of stresses. I mentioned earlier the difficulties connected with laboratory experiments of fracture. A much greater variability of materials and testing conditions in a laboratory often makes a comparison with

| Table 1. Fractal Exponents for Different Types of Condensed Matter Motion. |
|----------------|----------------|-------------|---------------|-------------|
| Phase space: | Size (M) | Time (T) | Space (R²) | Rotation (SO(3)) |
| Exponent: | β | θ | δ | k |
| Ideal solid material (no defects) | 0.5 | 0 | 2.0 | 0 |
| Regular solid material (small defects) | 0.5 | 0.5? | >2.0 (2.25–2.75) | >0 |
| Tectonic solid material (large defects) | 0.5 | 0.5 | 2.2 | 0.05–0.20 |
| Viscous fluids (large defects) | 0.5? | 1 – $\epsilon^*$ | 2.5–2.6 | ? |
| Galaxy clusters (large defects) | 0.5? | ? | 1.23 | ? |

* $\epsilon$ is a small positive quantity, $\epsilon \ll 1$. 
field results difficult and inconclusive; thus, attempts should made to obtain experimental data in a form comparable to that earthquake studies.

The conclusions from the studies of seismicity reported earlier, suggest that it should be possible to calculate the Earth’s stress patterns due to known earthquake faults and other defects and predict future seismicity on the basis of this information. A predicted earthquake occurrence may be compared to observations. Many obstacles need to be surmounted before such a development becomes possible. The total number of degrees of freedom in such numerical calculations is extremely large. The seismographic network in central California registers earthquakes with a magnitude close to zero; thus, the scale difference between the smallest and largest earthquakes is about 10^5. In 3-D, the total number of points amounts to 10^{12}. Discretization of time encounters similar problems: these small earthquakes have a characteristic time of the order of 10^{-2} s, whereas the time interval between very large earthquakes is hundreds and thousands of years (10^6-10^{10} s). Due to the intermittency of an earthquake process, most of the points both in space and time are “empty,” but even then we expect that computational difficulties due to an extremely large number of degrees of freedom are similar to those encountered in computations of turbulent flow for high Reynolds numbers. Even if the stress pattern has been computed, we need to know the earthquake fracture criterion to calculate the development of an earthquake rupture. This criterion should depend on all of the three invariants of the tensor. Furthermore, it is necessary to understand the coupling between brittle crust and plastic mantle. All these factors might make the Earth’s crust deformation calculations even more complex than the numerical computations of turbulent motion of fluid.

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