Multipole expansions of extended sources of elastic deformation

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SUMMARY

Seismic-moment tensors for composite point sources which consist of coherent and incoherent assemblages of elementary sources are examined. We estimate the sizes of the higher-order source components for several simple deterministic and stochastic models of the source region geometry, as well as for a model of stochastic fault geometry developed earlier by us, to simulate both the tensor and scalar geometrical properties and the temporal properties of complex faults and sources. We study the complexity of earthquake focal zones using expansions of seismic radiation of a finite source in vector spherical harmonics and/or in seismic-moment tensors of a rank higher than two. Five point elementary sources (one second-rank and four fourth-rank) are non-zero for planar earthquake faults; four possible other sources and zero for planar faults. Thus, the presence of seismic radiation from the latter sources can be used as an indicator of the complexity (non-planarity) of the source. If the source zone is almost planar, the number of free parameters contained in these sources can be greatly reduced, and their inversion from earthquake records becomes possible. Three classes of geometrical barriers (disclinations) corresponding to rotations around three nodal axes are identified for this condition. The strength of multipoles can be connected to particular geometric features of heterogeneous faults, i.e. the presence and strength of asperities, segmentation of a fault zone, like en échelon faulting, hence important tectonic results may be obtained from the inversions. Such a model, which involves barriers, requires a maximum of 20 degrees of freedom for its characterization.

Key words: Asperities, complexity of earthquake focal mechanism, finite sources of elastic waves, higher-rank seismic-moment tensors and their inversion, vector multipole expansions

1 INTRODUCTION

This paper is a continuation of our study of elementary point sources of elastic deformation. In the first paper of this series (Kagan 1987a) we considered general principles of classification of elastic point sources and, in particular, the static deformation caused by these sources. In a subsequent paper (Kagan 1987b) dynamic excitation functions of multipole point-sources have been examined. For brevity we will refer to those papers as H1 and H2, respectively. We use the mathematical notation of H1 and H2, supplementing it when necessary. As in the earlier papers, throughout this study we use the term rank, denoted by \( L \), for reference to the Cartesian tensors; the term order, denoted by \( I \), will be used only for vector spherical harmonics or for weight of the tensor representations of the 3-D rotation group.

In this paper we discuss finite sources of elastic deformation with a special emphasis on the study of complex earthquake focal zones and an inversion of seismic waves excited by these earthquakes. In our earlier paper (Kagan & Knopoff 1985a) we showed that, even if the extended earthquake focal zone consists of only pure double-couple (Burridge & Knopoff 1964) point elementary events (sources), the resulting total source, due to slight disorientation of these sub-events, must have a small component of the compensated linear vector dipole (CLVD) (Knopoff & Randall 1970). In Kagan & Knopoff (1985a) we also attempted to estimate the magnitude of this component. The results of these investigations indicate that for almost all of the shallow tectonic earthquakes, the strength of the CLVD component due to the complexity of the source, is very small. It does not seem probable that the size of the CLVD component can be used for characterization of the complexity of earthquake focal zones given the accuracy of modern inversions of the standard seismic-moment tensor. We also investigated the correlations of focal mechanism solutions (Kagan & Knopoff 1985b). We infer from the results of the above study that the earthquake focal zone is more complex and heterogeneous than the standard plane models of earthquake rupture. This complexity increases during the process of rupture, and should be manifested in properties of radiation of an individual event.

We initiated a further study to learn about the relationship between the geometry of faulting and
barrier/asperity strength, in order to study the dynamics of rupture of 3-D fault systems. Our purpose was to determine the degree of non-double couple radiation expected from non-planar faults, and subsequently to perform inversions from real data to determine the distribution of irregularities (Sipkin 1986). Since the barriers or asperities control the initiation and stopping of earthquake ruptures (Aki 1979; King & Yielding 1984), we had to develop a mathematical formalism to describe the asperities. We have shown (H1) that the complexity of the earthquake source-time function and hence the geometry of the focal zone should correspond to non-zero values of the third-rank moment tensor as well as higher-order terms in the multipolar expansions of elastic deformation.

The vector multipole formalism has been introduced in seismology largely through the work of Ben-Menahem & Singh (1968; 1981); for a more recent review of the field see, for example, Rybicki (1986). The advantage of this formalism is that the expansions of seismic radiation in the vector spherical harmonics is complete and orthonormal. Thus, if we know the appropriate vector multipoles, we can describe the whole field of seismic radiation, and only these multipoles can be determined (inverted) from the radiation data. The disadvantage of the multipoles, which possibly explains their relatively rare application in seismology, is the difficulty of relating them to physical, geological and tectonic parameters of the earthquake process.

One advantage of the seismic-moment tensor approach which has been developed by Aki (1966), Kostrov (1974), Gilbert & Dziewonski (1975), Backus & Mulcahy (1976), and Backus (1977a, b), is that the tensors are readily correlated both to geotectonic and geometric features of earthquake faults. This approach also easily incorporates specific conditions and constraints, i.e. the deviatoric nature of earthquake source, etc. These factors may have contributed to the dominance of this method in modern studies of seismic source. Recently there has been an increasing interest in using higher-rank seismic-moment tensors for inversion of seismic data (Silver 1983; Doornbos 1982a, b, 1984; Silver & Masuda 1985; Bukchin 1987; see also references in H1 and H2). However, the seismic-moment tensor method has its drawbacks, which were first pointed out by Backus & Mulcahy (1976), i.e. among sources corresponding to the fourth- and higher-rank seismic-moment tensors there are some which excite no seismic waves outside of a source zone. These sources, which we call null sources in H1, cannot be determined on the basis of seismic data. Our investigations (H1 and H2) show that the list of sources without observable elastic deformation should be expanded; in certain conditions, for example, not all of the sources which correspond to the seismic-moment tensor or the equivalent-force moment tensor of the third rank can be inverted from static deformation data. Seismic-moment expansions of all of the models of extended earthquake sources which are used in inversions of seismic data always contain high-rank tensors. The presence of null sources in the decompositions of these tensors indicates that such inversions are non-unique.

In H1 we defined symmetric, trace-free tensors (STFTs) as a vehicle enabling one to 'translate' the results obtained in vector multipoles into seismic-moment tensor form and the other way around. Thus, one can use the formalism best suited to a particular problem. To solve the problem of radiation from a finite complex seismic source, several methods, e.g. Green's function, potentials (Aki & Richards 1980; Ben-Menahem & Singh 1981), can be used for the forward problem. The use of the vector spherical harmonics and/or seismic-moment tensors for the solution of the forward problem may also be advantageous if one is interested only in long-period radiation. However, for the inverse problems only one method, that of vector multipoles, provides an appropriate formalism to obtain a unique solution; one should be able to analyse that solution and to transform it into the form which is more appropriate for a particular application.

Because the theory of elastic failure, especially in the heterogeneous initial condition, is not yet developed, we cannot employ a theoretical analysis for our purposes. What we can do at present is to calculate the higher-order seismic-moments for standard models of earthquake failure and then analyse elastic waves that these sources may excite in infinite homogeneous medium. We can also simulate the earthquake focal process (see Kagan & Knopoff 1981; Kagan 1982) and roughly estimate possible values of the sources other than the double-couple source, in a point approximation of the resulting complex finite source.

2 General Models of Extended Elastic Sources

2.1 Classification of moment tensors

In this subsection we review and organize different seismic-moment tensors we have examined in H1 and H2. We denote \( \mathbf{\Gamma} \) as a set of all seismic-moment stress glut tensors: \( \mathbf{\Gamma} \subset \mathbf{\Gamma} \) (read: \( \mathbf{\Gamma} \) is a member of \( \mathbf{\Gamma} \)). We denote \( \mathbf{\Phi} \) as a set of all of the equivalent-force tensors: \( \mathbf{\Phi} \subset \mathbf{\Phi} \), and a set of all tensor representations for sources which cause elastic deformation outside of a source zone is defined as \( \mathbf{E} \subset \mathbf{E} \). \( \mathbf{\forall} \) represents the set of vector spherical harmonics \( \mathbf{\forall} \subset \mathbf{\forall} \). General relations between these sets are described by

\[
\mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall},
\]

where \( \mathbf{\Phi} \subset \mathbf{\Gamma} \) means that \( \mathbf{\Phi} \) is a subset of \( \mathbf{\Gamma} \). In Table 1 relations between the tensors for indigenous sources are displayed for the tensors of the first four ranks. There is, for example, one-to-one (isomorphic) relation between all of the second-rank tensors; a similar relationship exists for third-rank dynamic sources (Backus & Mulcahy 1976). Thus the unique solution is, in principle, possible as a result of inversion for these sources. However, for the sources which are described by the fourth-rank tensor, the inversion is

<table>
<thead>
<tr>
<th>Rank</th>
<th>Static</th>
<th>General</th>
<th>Deviatoric</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
</tr>
<tr>
<td>3</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
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<tr>
<td>4</td>
<td>( \mathbf{\Gamma} \supseteq \mathbf{\Phi} \supseteq \mathbf{E} \supseteq \mathbf{\forall} )</td>
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</tr>
</tbody>
</table>

Table 1. Relationship between moment tensors
non-unique, i.e. we need additional assumptions or constraints to obtain a unique solution.

The symbol $\mathbb{H}$ is used for the set of all STF tensors (tensor representations of rotation group $SO(3)$) connected to the seismic-moment tensors: $\mathcal{H}, \mathcal{W} \in \mathbb{H}$. The symbol $\mathcal{F}$ is used for the set of all STF tensors connected to the equivalent force tensors: $\mathcal{F}, \mathcal{V}, \mathcal{D} \in \mathcal{F}$. These STF tensors are obtained from the seismic-moment tensors by use of appropriate contractions

$$\mathcal{F} = \phi(\mathcal{F}), \quad \mathbb{H} = \phi(\mathcal{F}),$$  

(2.2)

where $\phi$ denotes the contraction operator described for this case by equation (3.10–3.16) in H1.

The inverse relation between the seismic-moment tensors and the STFTs can be represented similarly:

$$\Phi = \psi(\mathcal{F}), \quad \Gamma = \psi(\mathbb{H}),$$  

(2.3)

where $\psi$ is the operator involving the tensor product of the STF tensors as well as the identity tensor, $\delta$, and the Levi-Civita tensor, $\varepsilon$, as described by (3.17–3.19) in H1. The STF tensors $\mathcal{W}, \mathcal{F}, \mathcal{V}, \mathcal{D}$ can be obtained from $\mathcal{H}$ by use of the linear operator $\chi$, which depends on the harmonic order number $l$, and on the Lame's constants $\lambda$ and $\mu$ (e.g. see equations 3.32–3.34 in H1 and A.13 in H2)

$$\mathcal{F} = \chi(\mathcal{H}), \quad \mathcal{D} = \chi(\mathcal{F}).$$  

(2.4)

Whereas the mappings (2.2) and (2.3) are isomorphic, mapping $\chi$ is not one-to-one, the number of degrees on the left-hand side of equation (2.4) is usually smaller than that on the right-hand side. The subsets of $\mathcal{F}$ or $\mathcal{H}$ tensors which map into the null-set of $\mathcal{D}$ constitute what we called in H1 null sources.

### 2.2 Extended sources

In our paper (Kagan & Knopoff 1985a) we considered composite or complex point-sources which are the result of two mathematical operations applied to elementary sources: the 3-D rotation and summation. Those are the minimum number of operations to yield a non-trivial point sources: the 3-D rotation and summation. Those are the subclasses of extended composite point-sources which are the result of three mathematical operations as simple as each other. Later we will also refer to extended sources formed by application of these three operations as simple sources. In this case the second-rank seismic-moment tensor density $\mathcal{g}_{ij}(x)$ is represented by

$$\mathcal{g}_{ij}(x) = \gamma(x) \mathcal{F}_{ij},$$  

(2.5)

where $\gamma$ is a scalar spatial function (Backus 1977a, p. 15). The finite source is then statistically equivalent to a scalar field; hence it is possible to find the centroid position, in regard to which the spatial third-rank seismic-moment tensor is zero (Backus 1977a). The two possible occasions when (2.5) is not satisfied are: (1) focal mechanism varies throughout the focal zone, i.e. it contains compression-dilatation centres and deviatoric source density in different proportions, or the CLVD content of a deviatoric source is changing in space and/or time; (2) constituent parts of the source are rotated (cf. Heaton 1982); in this paper we will consider only this type of 'non-simple' finite sources, i.e. the ones that do not satisfy (2.5).

Thus, for incoherent or disoriented extended composite sources the fourth mathematical operation has to be added: i.e. a 3-D rotation (disorientation). We will call these sources complex sources; they correspond to rotational dislocations, or disclinations (see also H1). For these sources the spatial centroid position corresponds not to a zero value of the third-rank tensor, but to a certain non-zero, in a certain sense minimal, value of the tensor (Backus 1977a). In both cases (simple and complex sources) the decomposition of the composite source shows that it contains elementary sources of many different classes and types. Further in this paper, we assume that all seismic-moment tensors are reduced to centroid-based coordinates.

We can create extended sources by application of the above operations to the elementary indigenous sources we have considered in H1 and H2. The simplest of these sources are compression-dilatation centres $\mathcal{H}^{(2)}$, they are invariant under 3-D rotation, and hence only operations which are valid for these sources are translations and scalar multiplications. Moment tensors for these sources belong to the class $\mathbb{C}$: $F \supseteq \mathbb{C}$. The class $\mathbb{D}$ of deviatoric sources, i.e. formed by deviatoric second-rank tensors $\mathcal{H}^{(3)}$, will be considered further in this paper; we will divide it into two subclasses: $\mathbb{D}'$, without rotation of elementary sources and $\mathbb{D}''$, with rotation. The numbers of degrees of freedom for a finite source zone formed by these sources are presented in Table 2, where for the sake of completeness we also include composite extended sources involving centres of rotation $\mathcal{V}$. The latter elementary internal sources are represented by third-rank tensors (H1). Using (2.2–2.4) we can obtain the STFT decomposition of above finite sources. In Table 3 we list the coefficients needed to calculate the average amplitudes, $\langle |u|^{2} \rangle$, of seismic waves produced by all sources which are described by seismic-moment tensors of fourth and lower ranks. $\eta$ above is the class of the source (see equations 2.5–2.6 in H2). To obtain the amplitudes we multiply the values in Table 3 by

$$\frac{|\mathcal{F}|}{4\pi r v} \cdot \left( \frac{\omega}{v} \right)^{l'},$$  

(2.6)

where $|\mathcal{F}|$ is the norm of the STF tensor shown in the second row of Table 3, $v$ is wave velocity, $r$ is distance, and $l'$ is shown in first row of Table 3 (see also 2.5–2.6 in H2). The STF tensor $\mathcal{D}$ in the table is given by (A.13) in H2; if we express it through the tensor $\mathcal{H}$, it equals

$$\mathcal{D} = - \mathcal{F} - \mathcal{H} \cdot \frac{l(2l-1)}{2l+1} + \mathcal{H} \cdot \frac{(l-1)}{2l+1}.$$  

(2.7)

Values of the STFT $\tilde{\mathcal{F}}$ are equal to $2 \cdot \mathcal{H}^{(3)}$ for $L = 3$ or to $2 \cdot \mathcal{H}^{(2)} + \mathcal{H}^{(4)}$ for $L = 4$ (see equation 3.14b in H1).
Table 2. Catalog of moment tensors of composite sources and numbers of their degrees of freedom

<table>
<thead>
<tr>
<th>Weight</th>
<th>Total number of degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Tensors of third rank

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>D'</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D''</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Tensors of fourth rank

<table>
<thead>
<tr>
<th>2</th>
<th>1</th>
<th>1</th>
<th>3</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>D'</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>D''</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>30</td>
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<tr>
<td>C</td>
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<td>0</td>
<td>0</td>
<td>6</td>
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<tr>
<td>V</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

The values for the second-rank source differ from those given by Aki & Richards (1980, problem 4.6) by a factor of \( \sqrt{2} \), since the tensor's norm in (2.6) is equal to \( M_0 \sqrt{2} \), where \( M_0 \) is the scalar seismic moment (see 3.4 in H2).

In our simulations of finite elastic sources we may use, in principle, both the equivalent-force nuclei (which are a set of vectors) and the stress glut nuclei (which are second-rank symmetric tensors), translating and rotating them through the 3-D space. If we apply the force nuclei which are equivalent to the stress glut nuclei, the simulation results should be the same. One problem we noticed in the simulations is the dependence of higher-rank moment tensors on the symmetry of forces constituting a nucleus of force. For example, the standard nucleus of compression–dilatation involves six forces distributed on vertices of a regular octahedron (or attached to the centres of faces of a cube). The extended source composed of such centres exhibits correct values for the second- and third-order STFTs, but yields a spurious fourth-order STFT. However, when we use a force distribution on the vertices of an icosahedron, both equivalent-force nuclei and stress-glut nuclei simulations become equivalent, at least, with regard to distribution of the STFTs up to a fourth order. Similar results have been obtained through numerical experiments with a tetrahedral distribution of forces in a force nucleus.

The above results can probably be explained by symmetry relations between vector spherical harmonics (or the STFTs) and symmetry groups of the 3-D rotation. We use symbols \( T, O, I \) to signify the symmetry groups of regular polyhedra: tetrahedral, octahedral and icosahedral symmetry, respectively. The symmetry group of the irreducible tensor representation (the STF tensors) \( F_{AI} \) or \( H_{AI} \) is taken as \( S_6 \). Following Ihrig & Golubitsky (1984; see also Gray 1976, p. 508)

\[ I \supset S_6 \supset O \supset S_5 \supset T \supset S_2, \] (2.8)

and this is the relation that we found in our simulations.

2.3 Temporal properties

In almost all of our previous discussion (H1 and H2) we only considered spatial moments of elastic sources. These moments are sufficient to obtain the solution of a static deformation problem. In our formulas for dynamic excitation functions (Appendix B in H1) we used the STF tensors \( F_{AI} \), which are functions of time \( t \) or frequency \( \omega \) without specifying the form of these tensor functions.

In regard to temporal properties of the sources, they can be classified as impulsive and continuous. We will not consider continuous sources here, since continuous sources which are of interest to seismology (like microseisms) are not concentrated in space, and so the multipole formalism is not useful to approximate such non-localized sources.

Table 3. Average amplitude of seismic waves for sources of various ranks \( L \) and orders \( l \)

<table>
<thead>
<tr>
<th>( \ell' )</th>
<th>( I-1 )</th>
<th>( I )</th>
<th>( I+1 )</th>
<th>( I+2 )</th>
<th>( I+3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>((L-1)N)</td>
<td>((L-2)N)</td>
<td>( \ell I )</td>
<td>((L-2)N)</td>
<td>((L-1)N) |</td>
</tr>
<tr>
<td>( L )</td>
<td>p/s</td>
<td>( \ell = L )</td>
<td>( \ell = L-1 )</td>
<td>( \ell = L-2 )</td>
<td>( \ell = L-3 )</td>
</tr>
<tr>
<td>2</td>
<td>p</td>
<td>3/8</td>
<td>–</td>
<td>1/4</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>1/8</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>p</td>
<td>5/16</td>
<td>–</td>
<td>1/5</td>
<td>1/5</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>1/4</td>
<td>3/8</td>
<td>1/4</td>
<td>1/4</td>
</tr>
<tr>
<td>4</td>
<td>p</td>
<td>9/32</td>
<td>–</td>
<td>3/8</td>
<td>1/5</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>1/8</td>
<td>3/16</td>
<td>1/10</td>
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<td>1/4</td>
<td>–</td>
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<tr>
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<td>3/16</td>
<td>1/10</td>
<td>1/10</td>
</tr>
</tbody>
</table>
However, for localized continuous stationary sources much of the following development could be used; we should only replace time functions of impulsive sources by appropriate correlation functions for continuous sources (cf. Doornbos 1984).

If the source action is instantaneous, the formulas of H2 are easily applicable, since then all tensor spectral functions are constants, i.e. simply tensors. We may sub-divide non-instantaneous sources into synchronous and non-synchronous sources. For synchronous source, i.e. sources for which displacement is time-coherent over total source zone, all spectral functions of all orders are identical: \( G_{(2)}(w) = G_{(2)}(w) = \cdots = G_{(n)}(w) \).

Expanding both for synchronous and non-synchronous sources, for example, \( G_{(4)}(w) \), in a Taylor series near \( w = 0 \), we have

\[
G_{(4)}(-i\omega) = \sum_{m=0}^{\infty} \Gamma_{(4)}^{(m)} \frac{(-i\omega)^m}{m!},
\]

where \( \Gamma_{(4)}^{(m)} \) terms are \( m \)th spectral derivatives of the spectral moment tensor. For time-localized sources we may interpret the spectral tensor moment as a characteristic function (Feller 1966) of a temporal distribution of seismic moment release. Then, for example, (see equation 3.6 in H1) we have

\[
\Gamma_{(n)}(w) = \int_{0}^{T} g(t, x) x^{-2} (t - t_0) m \, dx \, dt,
\]

where \( t_0 \) is the temporal centroid, and \( T \) is duration of the time rupture. Thus, we can give another interpretation of \( \Gamma_{(n)}^{(m)} \) terms as temporal moments of the seismic moment release (cf. Backus & Mulcahy 1976; Backus 1977a). As Backus (1977a) suggests, for earthquake sources we use a time derivative of \( g \) and \( \Gamma \) in (2.10). Using (2.9), it is easy to obtain for this case modifications of excitation formulae in H2.

Below we examine only sources which yield contributions in (2.9) up to the second order in \( \omega \) \((\omega^2)\) for a time-localized function of the seismic moment release. We refer to this finite-source approximation as \( \omega^2 \)-model (or approximation). As a result, in addition to the spatial moments we discussed earlier, for second-rank tensors we need to take into account temporal moments of the first- and second-degree; for the third-rank tensor only the first-degree temporal moment is needed (Backus 1977a). In Table 4, we calculate the total number of parameters defining such elastic sources. For example, for the most general composite source this number is a sum of (1) three temporal second-rank seismic-moment tensors, each having six degrees of freedom, (2) a centroid correction vector \( \xi \) with three degrees of freedom, (3) two temporal third-rank seismic-moment tensors with 18 degrees of freedom each, and (4) a fourth-rank tensor.

The numbers for the fourth-rank tensor which are listed in Table 4 refer to the number of parameters that can be inverted from seismic data. For the last two models in Table 4 this number is different for tensors \( \mathbf{E} \), \( \Phi \), and \( \Gamma \) (see Table 1); in the last line the numbers are 24, 30, and 36, respectively (H2). For the deviatoric model the numbers are 25, 30, and 30. Thus, for these models there are certain tensor components which either do not produce any displacement outside of the source zone (H1), or they are 'shielded' by other sources (H2, see also below). For impulsive sources, we should add to those degrees of freedom listed in Table 4 four parameters corresponding to the origin time and hypocentre-coordinates. These parameters are obtained on the basis of simple geometrical considerations. They can be used as initial estimates to calculate \( \Gamma_{(2)}^{(2)}, \Gamma_{(3)}^{(2)} \), and \( \xi \) (see Table 4).

### Table 4. Number of degrees of freedom for various source models

<table>
<thead>
<tr>
<th>Source model</th>
<th>( E_{(2)} )</th>
<th>( E_{(2)}^{(1)} )</th>
<th>( E_{(2)}^{(2)} )</th>
<th>( \zeta )</th>
<th>( E_{(3)} )</th>
<th>( E_{(3)}^{(1)} )</th>
<th>( E_{(3)}^{(2)} )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Double-couple:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point instantaneous</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Circular instantaneous</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Simple plane</td>
<td>4</td>
<td>1*</td>
<td>1</td>
<td>2*</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Complex plane</td>
<td>4</td>
<td>1*</td>
<td>1</td>
<td>3*</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>Simple volume</td>
<td>4</td>
<td>1*</td>
<td>1</td>
<td>3*</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>Complex volume</td>
<td>4</td>
<td>1*</td>
<td>1</td>
<td>3*</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td><strong>Non-double-couple:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complex deviatoric</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>3*</td>
<td>15</td>
<td>15</td>
<td>25(^*)</td>
<td>73(^*)</td>
</tr>
<tr>
<td>Complex general</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>3*</td>
<td>18</td>
<td>18</td>
<td>24(^*)</td>
<td>81(^*)</td>
</tr>
</tbody>
</table>

\( \xi \) is the spatial centroid vector (see eq. 4.26 in H1).

* Strictly speaking these parameters are not intrinsic degrees of freedom for these finite sources.

\(^{*}\) The number of free parameters are for \( \mathbf{E} \) tensor; the numbers for \( \Phi \) and \( \Gamma \) tensors see in Section 2.3.
which are 'shielded' by sources denoted by solid circles. Blank symbols correspond to sources connected by solid lines. Blank symbols correspond to sources which are 'shielded' by sources denoted by solid circles.

Figure 1. Summary plot of dynamic deviatoric elastic sources. Members of the same class for the equivalent-force moment tensors are connected by solid lines. Blank symbols correspond to sources which are 'shielded' by sources denoted by solid circles.

deviatoric; (2) moreover, microsources are double-couples; (3) the source zone is taken to be well-approximated by a planar fracture zone, i.e. the deviations from planarity are relatively small, second-order effects. Also below we only discuss seismic waves (dynamic excitation functions) in a far-field zone. An analysis of static extended sources or sources of near-field seismic radiation could be done in the same way.

In particular, if we accept condition (1), the catalogue of allowable elementary elastic sources varies from that shown in Figs. 1 of H1 and H2. In Fig. 1 (this paper) we display all of the deviatoric internal sources of dynamic elastic radiation. In the following discussions we limit ourselves to sources of the fourth-rank and lower; the total number of different elementary sources is nine. In Fig. 1, the first source from the left, marked \( F_{1,2m} \), is a familiar second-rank source with the quadrupole radiation pattern. For regular earthquake sources we assume that the radiation pattern is that of a double-couple, although recently there has been considerable interest in sources which have a significant CLVD component (Kagan & Knopoff 1985a; Sipkin 1986). The sources connected to the seismic-moment tensor of the third-rank (\( F_{2,2m} \) and \( F_{1,1m} \)) are equal to zero for finite seismic sources which do not involve non-trivial 3-D rotation of spatially different source components (see above). Thus, rotational dislocations (or disclinations) in an extended source zone should also be considered as an indicator of geometrical barriers or asperities encountered during an earthquake rupture (cf. Aki 1979; King & Yielding 1984). There are five different sources in the decomposition of the fourth-rank equivalent-force moment tensor. The quadrupole source shown by a blank circle is 'shielded' by a standard \( F_{3,1m} \) second-rank source; as we explained in H2, we cannot resolve this source without additional assumptions. Therefore, the total number of degrees of freedom of all of the sources contained in spatial seismic-moment tensors of the third- and fourth-rank is 40 (see Table 2); the total number of degrees of freedom for the time-localized source is 73 (see Table 4). The inversion of all of the above sources in case of a general extended elastic source therefore presents a formidable problem.

To further specify the sources we apply condition (2) above. A general, second-rank seismic-moment tensor has six degrees of freedom which we can enumerate, for instance, as three Euler angles specifying a system of coordinates, and three invariants of the tensor (cf. Kagan & Knopoff 1985a). The double-couple source is defined when the first and third invariants of the tensor are zero, thus it is specified by four parameters (see Table 4), only one of which (the norm of the tensor) is intrinsic. Condition (3) allows us to further restrict the space of possible finite elastic sources.

We take a circular Volterra dislocation, i.e. a dislocation with a uniform slip over the surface of the disc, as the simplest planar source zone. In equations 3.35 and 4.43 in H1, we list the values of the STFIs for this fault. (The last term in equation 4.43 should be read as \( \lambda_{32} = A/R_0^3 \)) These tensors (or their linear combinations) define sources of elastic deformation as detailed in H1 (Section 3) and H2 (Appendix A). Only five elementary sources (one second-rank and four fourth-rank) are non-zero for this fault; as we explained earlier, all of the third-rank sources are zero, as is the monopole \( (F_{1,0}^1) \) source. Later we shall refer to these five sources as standard earthquake sources.

In this section we use the system coordinates 'attached' to an earthquake fault. x-axis or 1-axis corresponds to the slip-vector, e (Ben-Menahem & Singh 1981, p. 186, Fig. 4.24), y-axis is normal to the fault-plane, n; z-axis corresponds to the null-vector, b. The fault-plane is hence xz- or (13)-plane.

3.1 Simple planar sources

Dependence of amplitudes of shear waves on the angular frequency \( \omega \) for the circular fault is displayed in Fig. 2. Frequency values are scaled so that the value of the 'corner' frequency \( (\omega_c = \beta/R_0) \) where \( \beta \) is the velocity of shear waves in a medium, and \( R_0 \) is the radius of the circle), corresponds to abcissa 1.0 in the plot. We also normalized...
the amplitudes so that the amplitude of a shear wave from a double-couple source is equal to unity. Since the second source term in equation B.1c of H2 which corresponds to the fourth-rank blank circle in Fig. 1 has the sign opposite to that of the first term, the total amplitude of the quadrupole radiation decreases as the frequency increases. Using the values of source strengths in equation (4.43) of H1 and the average amplitude values from Table 3, we compute the radiation caused by both sources. The amplitudes are identical for the frequency value
\[
\omega = \frac{B}{R_0} \cdot 2 \left( \frac{10}{3} \right)^{1/2} = 2 \left( \frac{10}{3} \right)^{1/2} \omega_{cr}.
\] (3.1)

In reality the amplitude value of the first quadrupole radiation is non-zero for this frequency, since radiation caused by higher-level (see H2, Section 3.3) STFT terms should be significant for relatively high values of frequency. To estimate the latter radiation level we need to calculate seismic-moment tensors of the fifth-rank and higher; this has not been done in this work. By matching the shape of the spectrum of quadrupole radiation of an earthquake to curve 1 in Fig. 2, we can determine the strain drop and the effective radius of an equivalent circular Volterra dislocation.

In normal seismological practice, two parameters are used to characterize an earthquake source: the scalar seismic moment and corner frequency, which in turn define stress drop and the size of the fault (Aki & Richards 1980). Determination of the size of earthquake fault on the basis of corner frequency depends on a model (Silver 1983; Doornbos 1984 and references therein) and on the reliable registration of the high-frequency part of earthquake radiation. If the earthquake focal zone has a heterogeneous structure, standard methods of determination of the corner frequency may yield highly biased estimates (H2).

The most complicated model of planar earthquake fault involves four degrees of freedom: seismic-moment surface density μ, and three free parameters corresponding to the moments of inertia (Silver 1983; see also Backus 1977a) of the planar fault patch. One way to resolve the inversion problem for such an extended source is to express excitation functions directly through those parameters of the fault, and then to obtain the estimates of the parameters by inversion of real radiation data (Doornbos 1982a; Silver & Masuda 1985). A clear advantage of such an approach is its simplicity and straightforwardness. However, we cannot be sure that our solution is optimal; if we want to test a more complicated model, we need to start our new solution from scratch.

As another inversion method, we determine effective parameters of the fault zone using the orthogonality property of the vector spherical harmonics; hence the influence of other multipoles can be 'filtered out', for example, by appropriate stacking of seismogram records (Gilbert & Dziewonski 1975; Buland, Berger & Gilbert 1979). Here we do not need to commit ourselves to a particular model of source zone. Moreover, we can be assured that all the pertinent spectral information (in the \(\omega^2\)-approximation, for example), available in seismograms, has been extracted; thus this method is more universal. However, this approach requires, in most cases, a large number of well-distributed seismographs which surround a source zone. As (2.8) suggests, the minimum number of such seismographs for registration of a radiation of the fourth-order harmonics should be 12, distributed at vertices of an icosahedron (cf. Agnew et al. 1986). If, as expected, the distribution of seismographs does not satisfy the above conditions, estimates of the strength of multipoles should be correlated, and hence they are not orthogonal. Perhaps the final choice of any of the discussed methods depends on the particulars of each case.

As an illustration, we take a planar strike-slip fault in a parallelogram shape, which is in the \(xz\) plane (Fig. 3a). We take fault \(2a \times 2b\) with the \(a\) side parallel to the slip vector. Since there is no 3-D rotation, the third-rank tensor is zero for such a fault. Non-zero components of the fourth-rank tensor are
\[
\begin{align*}
\Gamma_{1211} &= \Gamma_{2111} = \frac{3}{4} abm(\alpha^2 + \beta^2), \\
\Gamma_{1233} &= \Gamma_{2133} = \frac{3}{2} abm(2\beta^2 - \alpha^2 - \beta^2), \\
\Gamma_{1233} &= \Gamma_{2133} = \frac{2}{3} abm(2\beta^2 - \alpha^2 - \beta^2), \\
\Gamma_{1213} &= \Gamma_{2113} = \frac{1}{3} ab^2 mp. 
\end{align*}
\] (3.2)

For non-zero components of the symmetric trace-free tensors (STFTs) we obtain
\[
\begin{align*}
\mathcal{H}_{1112} &= \frac{1}{2} abm(4a^2 + 4p^2 - \beta^2), \\
\mathcal{H}_{1222} &= -\frac{3}{2} abm(3a^2 + 3p^2 + \beta^2), \\
\mathcal{H}_{1233} &= \frac{2}{3} abm(2\beta^2 - \alpha^2 - \beta^2), \\
\mathcal{H}_{1233} &= \frac{1}{3} ab^2 mp. 
\end{align*}
\] (3.3)

\[\begin{align*}
2\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{1}{3} ab^2 mp, \\
2\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{1}{3} ab^2 mp, \\
3\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{3}{2} ab^2 mp, \\
3\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{3}{2} ab^2 mp, \\
\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{3}{2} ab^2 mp, \\
\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{3}{2} ab^2 mp, \\
\mathcal{H}_{12} &= \frac{3}{2} abm(a^2 + \beta^2), & \mathcal{H}_{11} &= -\frac{3}{2} ab^2 mp.
\end{align*}\]

(a)

(b)

(c)

Figure 3. Schematic diagram of earthquake faults: (a) parallelogram fault, (b) bent fault, (c) en échelon fault.
All of the parameters of a parallelogram plane fault can be obtained by an inversion of the last four components. It means that, in principle, we need to invert only dipole and quadrupole radiation of a planar earthquake fault to obtain all of its parameters. The planar fault could be, of course, parameterized in other forms than the parallelogram shape used here. For example, an ellipse with axes 2a and 2b, the major axis at an angle \( \alpha \) with the slip-vector, is another parameterization which also requires four degrees of freedom. We do not present these computations here.

From the above formulas we can easily obtain expressions for norms of the STFTs

\[
|\mathcal{F}^{(4)}_{(2)}| = \frac{4\sqrt{2}}{3\sqrt{3}} \, abm \sqrt{2a^4 - a^2 b^2 + 4a^2 p^2 + b^4 + 3b^2 p^2 + 2p^4},
\]

\[
|\mathcal{F}^{(4)}_{(3)}| = \frac{4\sqrt{2}}{3\sqrt{15}} \, abm \sqrt{2a^4 - 5a^2 b^2 + 4a^2 p^2 + 5b^4 + 7b^2 p^2 + 2p^4},
\]

\[
|\mathcal{F}^{(4)}_{(2)}| = 0, \quad |\mathcal{F}^{(4)}_{(2)}| = \frac{2\sqrt{2}}{3} \, abm \sqrt{a^2 b^2 + 2a^2 p^2 + b^2 p^2 + p^4},
\]

\[
|\mathcal{F}^{(4)}_{(1)}| = \frac{2}{3} \, abm (a^2 + b^2 + p^2),
\]

\[
|\mathcal{F}^{(4)}_{(1)}| = \frac{4}{3} \, abm \sqrt{a^4 + 2a^2 p^2 + b^2 p^2 + p^4},
\]

\[
|\mathcal{F}^{(4)}_{(0)}| = 0, \quad \text{and} \quad |\mathcal{F}^{(4)}_{(0)}| = 0,
\]

where the norm of the second-rank tensor is equal \( |\mathcal{F}^{(2)}_{(3)}| = 4\sqrt{2}abm \). The norm of the tensor \( \mathcal{F}^{(4)}_{(2)} \) (see equations 3.12b and 4.37b in H1 and 3.14 in H2) which we use for the normalization purposes, is

\[
|\mathcal{F}^{(4)}_{(2)}| = \left[ 2 \cdot |\mathcal{F}^{(4)}_{(2)}| + 3 |\mathcal{F}^{(4)}_{(2)}| \right] \\
= \frac{4\sqrt{2}}{3} \, abm \sqrt{4a^4 + 4a^2 b^2 + 8a^2 p^2 + b^4 + 3b^2 p^2 + 4p^4}. \quad (3.5)
\]

The above values of source strengths together with entries in Table 3 (see equations 2.6–2.7) give an opportunity to calculate the average amplitude of seismic waves caused by these sources.

As we mentioned above, by taking \( p = 0 \) and \( a = b \), we obtain formulae for a square fault. These expressions can be compared to (3.35) and (4.43) of H1. Since in H1 we use a different fault-plane orientation, coordinate axes are to be substituted \( 1 \to 3, \; 2 \to 1, \; 3 \to 2; \) the Levi–Civita tensor for this substitution \( \epsilon_{312} = 1 \) (see A.4 in H1), thus the sign of toroidal ('odd') tensor components remains the same. Using (3.35) and (4.43) of H1, we calculate an effective radius of a circular fault which has the same quadrupolar radiation as a general planar fault:

\[
R_{\text{eff}} = \left[ \frac{4 \cdot |\mathcal{F}^{(4)}_{(2)}|}{3 \cdot |\mathcal{F}^{(4)}_{(2)}|} \right]^{1/2}, \quad (3.6)
\]

so that for the circular fault \( R_{\text{eff}} = R_0 \).

In Table 5 we list the values of scaled norms of the STF tensors for several sub-types of the planar model of an earthquake fault (cf. Backus 1977a, b). For normalization the norms have been divided by the value of

\[
R_{\text{eff}}^2 |\mathcal{F}^{(2)}_{(3)}| = \frac{4}{3} \cdot |\mathcal{F}^{(4)}_{(2)}|, \quad (3.7)
\]

(see 3.6). By doing so, we have essentially reduced all fault

| Table 5. Strengths of normalized standard STF seismic moment tensors for plane sources |
|---|---|---|
| **Deterministic sources** | **Stochastic sources** |
| **line** | **square** | **min** | **average** |
| **line** | **a < b** | **a = b** | **a > b** |
| \( |\mathcal{K}^{(4)}_{(4)}| \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \)
| \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \) | \( \frac{2}{\sqrt{1}} \)
| \( |\mathcal{K}^{(4)}_{(1)}| \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \)
| \( |\mathcal{K}^{(4)}_{(2)}| \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \)
| \( |\mathcal{K}^{(4)}_{(3)}| \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \)
| \( |\mathcal{K}^{(4)}_{(4)}| \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \) | \( \frac{1}{\sqrt{1}} \)

* \( b = \frac{\sqrt{6}}{\sqrt{3}} a \) for \( |\mathcal{K}^{(4)}_{(4)}| \)_{\text{min}}; \( b = \frac{\sqrt{14}}{\sqrt{3}} a \) for \( |\mathcal{K}^{(4)}_{(3)}| \)_{\text{min}}; the parameter \( p = 0 \) in both cases.
models to a standard size. We deduce, for example, that with regard to elementary sources of second and fourth ranks, a square fault produces the same radiation pattern as a circular fault. The effective radius of the square fault is:

\( R_{eff} = 2a/\sqrt{3} \).

We denote these normalized STF tensors by the symbol \( \mathcal{F}_A \). For most of the sources, linear fault models represent extreme cases (see Table 5).

As was mentioned above, if we substitute \( x \) axis by \( y \) axis, the toroidal dipolar radiation changes its sign; hence we can resolve the fault-plane ambiguity (cf. Backus, 1977a, p. 19) if we know the dipole's parameters. Similarly, if \( p \neq 0 \), the axis of the toroidal oscillations is in the fault-plane, so the ambiguity can again be resolved. In general, if we are able to determine the amplitudes of higher-order harmonics (\( \psi_p^{(0)}(\omega) \), \( \psi_p^{(4)}(\omega) \) and \( \psi_p^{(8)}(\omega) \) waves), we should be able to resolve the fault-plane ambiguity (see 4.43 in Hl). Backus (1977a, pp. 19–20) regards it as an 'astonishing and fortunate' result that the components of the equivalent-force moment, which are needed to resolve the ambiguity, have a one-to-one relation (see 2.4) to corresponding tensors \( \mathcal{F}_G \) (see Fig. 1 in Hl).

Let us now consider an asymmetric rupture on a plane, such as unilateral propagation of the fracture. We examine the third-rank seismic-moment tensor in the spatial centroid coordinates, which correspond to the final size of the earthquake fault. In this coordinate system the third-rank tensor is non-zero in the early stages of rupture. If we take components of the tensor as

\[ I_{121} = I_{121} = A, \quad I_{123} = I_{123} = B, \]

non-zero values of the STFTs are

\[ \mathcal{F}^{(3)}_{111} = \frac{8}{15} A, \quad \mathcal{F}^{(3)}_{123} = \frac{B}{3}, \]

\[ \mathcal{F}^{(3)}_{222} = -\frac{2}{5} A, \quad \mathcal{F}^{(3)}_{333} = -\frac{2}{15} A, \]

\[ \mathcal{F}^{(3)}_{131} = -\frac{A}{2}, \quad \mathcal{F}^{(3)}_{313} = -\frac{B}{2} = B, \]

and

\[ 2\mathcal{F}^{(3)}_{113} = A. \]

Toroidal source \( \mathcal{F}^{(3)} \) is sufficient to obtain all components of the third-rank 'directivity' tensor. Calculation of total radiation caused by all of these (see 2.10) sources yields a dipolar directivity pattern (Doornbos 1982b; Silver, 1983). For a planar earthquake source (or any source satisfying 2.5) \( I^{(1)}_{\alpha} \) is the temporal centroid \( t_0 \); in time domain, \( I^{(2)}_{\alpha} \) is proportional to the square of the width of a pulse (Silver & Masuda 1985).

In Table 4 we list numbers of degrees of freedom for various sources corresponding to a 'simple planar' fault model. To complete the picture, a 'simple volume' model is also included in the table. The model is a distribution of parallel double-couple elementary sources extended in all three orthogonal directions (cf. Doornbos 1982a).

### 3.2 Complex planar sources

By complex (incoherent) planar sources we mean earthquake planar fault zones, where dislocations which comprise the finite source might undergo infinitesimal (or small in some sense) 3-D rotations (disorientation). Rotations are considered infinitesimal since in this case they are commutative, i.e. the final result of a sequence of rotations does not depend on the order in which the rotations are executed. The third-rank seismic-moment tensor which is, in general, non-zero in this case, can be represented as a sum of six components:

\[ I_{(3)} = \sum I_{(3)} + \sum I_{(3)} + \sum I_{(3)} + \sum I_{(3)} + \sum I_{(3)}, \]

where the left superscript denotes the axis of rotation, and the left subscript shows the integration axis, e.g.

\[ \sum I_{(3)} = 2 \int \mathbf{x} \mathbf{g}(x, z) \phi(x, z) \, dx \, dA = \langle 2x \mathbf{g} \phi \rangle = \beta_x. \]

Here, \( A \) signifies the vector of fault-plane coordinates \( (x, z) \), \( \phi_x \) is an \( x \)-rotation angle (\( \phi_x \ll 1 \));

\[ \mathbf{g}_y(x) = \gamma(x)(\vec{I}_{ij} - \delta_{ij}), \]

(see 2.5) and, in its turn,

\[ \vec{I}_{ij} = R_{ij} \mathcal{I}_{pq} R_{ij}, \]

where \( R_{ij} \) is an infinitesimal, normalized rotation matrix. The factor 2 in (3.11) enters the formula since the rotation transformation of the second-rank tensor (3.13) involves \( \sin 2\phi \) (see 3.23 below), which for small angles is well approximated by 2\( \phi \).

Similarly,

\[ \gamma_x = \langle 2x \mathbf{g} \phi \rangle. \]

The formulae (3.11–3.14) mean that \( \beta_x \) and \( \gamma_x \) are proportional to coordinates of the centre of mass for the product \( (x \psi_0) \). Non-zero values for \( \beta_x \), for instance, means that average \( \psi_0 \) has opposite signs for rotations on +\( x \) and −\( x \) parts of the fault-plane, i.e. the rotation is antisymmetric with regard to the \( x \)-axis. The formulae for the \( y \)-rotation are analogous to (3.11), whereas, for the \( z \)-rotation the factor 2 is to be replaced by the factor 4 in (3.11). Thus, values of components of the third-rank tensor due to disorientation around the null vector are doubled as compared to the rotation around other axes.

#### 3.2.1 \( x \)-rotation

This transformation, which uses a slip vector as an axis of rotation, creates geometrical barriers/asperities which King & Yielding (1984, see their Fig. 11) call conservative barriers. Non-zero values of the third-rank seismic-moment tensor are

\[ \sum I_{(3)} = \sum I_{(3)} = \beta_x, \quad \sum I_{(3)} = \sum I_{(3)} = \gamma_x. \]

Non-zero values of the STFTs are

\[ \mathcal{F}^{(3)}_{111} = \frac{2}{5} \gamma, \quad \mathcal{F}^{(3)}_{122} = -\frac{2}{15} \gamma, \quad \mathcal{F}^{(3)}_{333} = \frac{8}{15} \gamma, \]

\[ \mathcal{F}^{(3)}_{131} = \frac{8}{15} \beta, \quad \mathcal{F}^{(3)}_{313} = -\frac{2}{15} \beta, \quad \mathcal{F}^{(3)}_{333} = -\frac{2}{5} \beta, \]

\[ \mathcal{F}^{(3)}_{122} = -\frac{\gamma}{2}, \quad \mathcal{F}^{(3)}_{222} = \frac{\gamma}{2}, \quad 2\mathcal{F}^{(3)}_{113} = \gamma, \quad 2\mathcal{F}^{(3)}_{131} = \beta. \]

Only one new source, a monopole, appears in a
decomposition of the fourth-rank seismic-moment tensor:

\[ \mathcal{M}_0^{(4)} = 2 \Gamma_{(13)(13)} = 2 \int_A xz \bar{g}(x, z) \phi_a(x, z) \, dA \]

\[ = \left\langle 4xz \bar{g} \phi_a \right\rangle, \quad (3.17) \]

where subscripts in parentheses mean symmetrization of a tensor with regard to those subscripts (see A.2a in H1).

3.2.2 \textit{y-rotation}

\textit{y-axis} corresponds to the vector normal, \( \mathbf{n} \), to the fault plane, which means that the infinitesimal rotations are in the fault plane. We obtain

\[ \varepsilon \Gamma_{(y)} = \gamma \Gamma_{(y)} = \beta_y, \quad \varepsilon \Gamma_{(y)} = \gamma \Gamma_{(y)} = \gamma. \quad (3.18) \]

Non-zero values of the STFTs are

\[ \mathcal{M}_1^{(3)} = - \frac{2}{15} \gamma, \quad \mathcal{M}_2^{(3)} = \frac{\beta}{3}, \]

\[ \mathcal{M}_3^{(3)} = \frac{\beta}{5} \gamma, \quad \mathcal{M}_4^{(3)} = \frac{8}{15} \gamma. \quad (3.19) \]

\[ \mathcal{M}_5^{(3)} = \frac{\gamma}{2}, \quad \mathcal{M}_6^{(3)} = - \frac{\beta}{5} \gamma, \quad \mathcal{M}_7^{(3)} = \frac{\beta}{2}, \quad \text{and} \quad \mathcal{M}_8^{(3)} = \beta. \]

The above formulae are equivalent to (3.8–3.9) if we substitute subscript 1 by 3. The toroidal source changes sign as the result of such substitution. The monopole source is zero for this rotation case.

3.2.3 \textit{z-rotation}

\textit{z-axis} corresponds to the null vector of the focal mechanism. Thus, this rotation creates barriers/asperities which King & Yielding (1984) call non-conservative geometrical barriers. We have

\[ \varepsilon \Gamma_{(z)} = \gamma \Gamma_{(z)} = \beta_z, \quad \varepsilon \Gamma_{(z)} = \gamma \Gamma_{(z)} = \gamma. \quad (3.20) \]

Non-zero values of the STFTs are

\[ \mathcal{M}_1^{(3)} = \frac{3}{5} \beta, \quad \mathcal{M}_2^{(3)} = \frac{\gamma}{3}, \]

\[ \mathcal{M}_3^{(3)} = - \frac{7}{15} \beta, \quad \mathcal{M}_4^{(3)} = - \frac{2}{15} \beta, \]

\[ \mathcal{M}_5^{(3)} = - \gamma, \quad \mathcal{M}_6^{(3)} = \frac{\beta}{2}, \quad \text{and} \quad \mathcal{M}_7^{(3)} = \beta. \quad (3.21) \]

The monopole source is

\[ \mathcal{M}_0^{(4)} = \Gamma_{1111} = - \Gamma_{2222} = \frac{8}{15} A, \quad \mathcal{M}_0^{(4)} = \Gamma_{2222} = \sqrt{2} A, \quad (3.22) \]

The strengths of the sources are

\[ |\mathcal{M}_0^{(4)}| = \frac{8}{15} A, \quad |\mathcal{M}_0^{(4)}| = \sqrt{2} A, \quad (3.24) \]

The fourth-rank moment tensor for the bent fault (Fig. 3b) is given by (see equation 3.2)

\[ \Gamma_{1111} = \Gamma_{2222} = \frac{3}{2} ab \cos \alpha, \quad \Gamma_{1111} = \Gamma_{2222} = \frac{3}{2} ab \cos \alpha, \quad \Gamma_{1111} = \Gamma_{2222} = \frac{3}{2} ab \cos \alpha, \quad (3.25) \]

\[ \text{The values of the components of the STFTs can be easily calculated from (3.25); for small values of the angle} \quad \alpha, \quad \text{the values of the STFT components are equal to that for} \quad \alpha = 0 \quad \text{multiplied by a factor of the order} \quad \cos \alpha. \]

In Fig. 4 amplitudes of shear waves excited by the bent square fault are displayed. Amplitudes of waves caused by the third-rank sources are so high that they modify the radiation pattern for this fault significantly. Since third-rank amplitudes are proportional to frequency, for \( \omega = 2\alpha \), the amplitude of the third-rank radiation is only slightly smaller than that of the quadrupole radiation of the plane fault. Thus, near and beyond the corner frequency, the resulting radiation could easily be interpreted as belonging, for example, to a source with a significant CLVD content, or as
the standard double-couple seismic source which also has a non-zero isotropic radiation. Radiation patterns caused by small rotations of focal mechanisms (Section 3.2) are similar to that displayed in Fig. 4. However, as we discuss later, a strength of the ‘third-rank’ multipoles for real earthquakes is, in most cases, much smaller than that shown in Fig. 4.

Next we consider the fault which has \( q = \text{échelon} \) pattern (Fig. 3c). The third-rank seismic moment tensor is zero. The fourth-rank seismic moment tensor for the fault is given by

\[
\begin{align*}
\Gamma_{1111} &= \Gamma_{2111} = \frac{4}{3}a^3bh'm, \\
\Gamma_{1212} &= \Gamma_{2121} = \Gamma_{1221} = \Gamma_{1121} = -2a^2bh'm, \\
\Gamma_{1222} &= \Gamma_{2122} = 4abh^2m.
\end{align*}
\]

(3.26)

Again, for small values of \( h \), the values of the STFTs are almost the same as shown in equations 3.3 and 3.4 for the case of a rectangular continuous fault. The major difference is the value of the monopole source \( |\mathcal{H}(0)| \) which is equal to \(-4a^2bh'm\) in the latter case (cf. 3.4).

As we mentioned in H2, the presence of this isotropic radiation pattern may constitute an additional difficulty in the problem of the discrimination of explosions from complex earthquakes. Our calculations show that if \( a = h\sqrt{3} \) (see Fig. 3c) the amplitude of P-wave, excited by this source at the ‘corner’ frequency, is about 7 per cent of the maximum amplitude of P-wave caused by a double-couple source. Since the amplitude of P-wave for this source increases as \( \omega^2 \), it may be comparable to a ‘double-couple’ amplitude for relatively large-frequency values, especially near nodal planes of the latter wave. We may use the strength of the monopole source to describe the effective ratio \( h/a \) for complex earthquake faults.

4 SIMULATION OF EARTHQUAKE FAULTS

We have constructed a stochastic self-similar model of earthquake occurrence that simulates well all of the documented statistical information regarding regional and world-wide seismicity: the magnitude-frequency law, the magnitude-coda law, the law for the temporal distribution of aftershocks and the spatial distribution of fractures on a fault system (Kagan & Knopoff 1981; Kagan 1982). This model involves a critical branching process in which a nearest-neighbour selection is made from a relationship between unit earthquakes. One unit event triggers the occurrence of a second at a random time. The random time has a strong weighting factor built into it that favours earlier times over later ones. When a second unit earthquake occurs, its location is determined by the statistical spatial selection formula that uses a rotational Cauchy distribution. One unit parent may have more than one unit offspring.

Although our stochastic kinematic model seems to reproduce both short- and long-term properties of earthquake rupture, we have been able to test it only for more long-term effects (Kagan & Knopoff 1981; Kagan 1982). Since the model is based on quasi-static concepts, it is quite possible that the short-period radiation is not a faithful representation of seismic waves of real earthquakes. As we have noted earlier (Kagan & Knopoff 1985a, b), the model has some other drawbacks: rotations of elementary dislocations do not take into account the fault-plane symmetry, which means that this model cannot be used to test which of the random disorientations, discussed in Section 3.2, are preferred in natural earthquake faults. However, this is the only model that allows us to realistically represent non-planarity of earthquake faults as well as disorientation and complexity of the fault zone.

We have created a number of synthetic faults. The value of the parameter \( \phi_0 \) which controls the degree of branching was set to \( 5 \times 10^{-6} \) and to \( 5 \times 10^{-5} \). As discussed in Kagan (1982), these values of \( \phi_0 \) correspond to the parameter limits for the degree of branching or non-planarity of real earthquake faults. To test whether high values of \( \phi_0 \) contribute to violation of scale-invariance of the synthetic earthquake fault (Kagan & Knopoff 1985a), a simulation with \( \phi_0 = 10^{-10} \) has been also executed. We determined the STFTs for the synthetic faults, and found statistical distributions of norms of these tensors. These distributions represent a compact picture of a complex earthquake source. Using these distributions we can calculate all of the possible radiation patterns of tectonic earthquakes. We see that normalized values for the standard STFTs are distributed over a relatively narrow range (see Table 5). Most of the distributions for the planar fault source are concentrated closer to a circular (or square) form.

Synthetic faults, as expected, yielded non-zero values of non-standard sources, giving evidence of their complexity. As anticipated, there is no real difference in distribution of strengths for all six types of asperities for synthetic fault; the value of the strength for z-rotation is twice as large as those for x- and y-rotation (see Section 3.2 for the explanation). Cumulative distributions of strengths for two non-standard sources are shown in Fig. 5. To normalize the strengths of the STFTs we used \( R_{\text{eff}} \cdot |\mathcal{H}| \) (see 3.6, 3.7) for \( \mathcal{H}^{(3)} \) and \( R_{\text{eff}} \cdot |\mathcal{H}| \) for the third-rank tensor:

\[
\psi^{(3)} = \frac{\Gamma_{1111}}{R_{\text{eff}} \cdot |\mathcal{H}|^{(2)}}.
\]

(4.1)
The tail of the distribution for both variables is almost identical to the 1-D distribution, deteriorating as the value $G_0$ increases (Fig. 5).

Hence the values of $\Psi_{111}$ show a scaled strength of non-conservative barriers (see Sections 3.2.3 and 3.3), and $1\mathcal{X}_0^{(1)}$ is the normalized strength of an isotropic (monopole) source. $|\Psi_{111}| = 0.1$ approximately corresponds to a bent fault (Fig. 3c) with $\alpha = 10^\circ$ (see Section 3.3 and Fig. 4).

These distributions belong to a class of stable, or self-similar, distributions; they are described by a power-law strength values for these sources concentrate strongly near zero. However, if we take $\phi_0 = 10^{-10}$ the resulting distribution for both variables is almost identical to the 1-D Cauchy distribution with the cumulative function (Feller 1966)

$$F(x) = \frac{2}{\pi} \arctan \frac{x}{t}.$$  \hspace{1cm} (4.2)

where $\infty \geq x \geq 0$, and $t = 1.6 \times 10^{-4}$ for $\Psi_{111}$. The tail of the distribution is linear in the log–log plot. The correspondence between simulated distributions and the Cauchy distribution deteriorates as the value $\phi_0$ increases (Fig. 5), but as we suggested earlier (Kagan & Knopoff 1985a, b) the strength values for these sources concentrate strongly near zero.

Our results presented in Fig. 5 indicate that only about 1 per cent (or less) of the earthquake focal zones are complex enough to excite these non-standard waves with an amplitude exceeding 1 per cent of the amplitude of double-couple radiation near or at the 'corner' frequency (see Table 3 and Fig. 4). However, since radiation from these sources has an $\omega$ or $\omega^2$ dependence, an amplitude of the waves excited by them might be significant for frequencies far beyond $\omega_c$. 

5 DISCUSSION

In Table 4 two simple models are shown (upper two rows), which are now used predominantly in the analysis of source region (Aki & Richards 1980; Ben-Menahem & Singh 1981). Recently, more complicated source models, like ‘simple plane’ shown in Table 4, have been explored by Silver & Masuda (1985) in interpretations of seismic radiation of earthquake focal zone. It is easy to fill the ‘gap’ in models between ‘circular instantaneous’ and ‘simple plane’ by plane models exhibiting increasing number of degrees of freedom (see e.g. Doornbos 1982a; Silver 1983 and references therein). However, some of these models may map themselves into a subset of parameters which could be identical for various models. Thus, such models are indistinguishable from the point of view of the $\omega^2$-approximation. Moreover, in many cases seismic signals of earthquakes are long-period band-limited, the $\omega^2$-model may be the only appropriate approximation for the description of seismic-wave spectra.

In this paper, we have concentrated our efforts on expanding the model ‘space’ towards more complex models involving asperities or barriers. A standard definition of barriers/asperities usually involves an assumption of stress or strength concentrations or heterogeneities on a planar earthquake fault. Indications of the presence of asperities are usually seen in strong variations of seismic-moment release on various patches of an earthquake fault. Usually the proof of such stress/strength concentrations can be obtained from data only on the basis of indirect inferences or hypotheses. In our definition, geometrical barriers (or asperities) are, in principle, directly measurable quantities – they are the strengths of multipoles connected to the third-rank seismic-moment tensor; we also identify them with disclinations of solid state physics. Thus our definition is a constructive definition.

Our simulations seem to show that for most of the earthquake sources, values of components of a third-rank tensor should be very small, perhaps, below the detection threshold for almost all earthquakes. However, as noted in Section 4, we cannot be sure that the stochastic model we used to estimate the strength of asperities is a faithful representation of real earthquake faulting in this respect. We may hypothesize that in order for an earthquake fault to stop propagation, the earthquake source must have some non-double-couple component which effectively ‘locks’ the fault zone of the earthquake. If this assumption is correct, then each completed earthquake has to have a non-zero third-rank tensor measuring the strength of ‘barriers’ which stopped it. Then, the distribution of the magnitude of the third-rank tensor instead of being monomodal with the mode corresponding to a zero average should be bimodal, so that the average is still equal to zero, but high concentrations of the strength are situated on both positive and negative sides of the distribution.

If the above assumptions are correct, then the proper $\omega^2$-model of earthquake source is that listed as ‘complex plane’ in Table 4. We do not know whether all three types of barriers examined in Section 3.2 are present in real earthquake faults. On the basis of simulations described in Section 4 we might try to estimate the validity of a ‘complex plane’ approximation. This approximation will fail if at least two rotations are significantly non-zero. In this case, even if constitutive parts of a focal zone are double-couples, the resulting extended deviatoric source should contain a CLVD component (Kagan & Knopoff 1985a). If we assume that the rotations are independent, then only one tectonic earthquake in $10^4-10^5$ should have such a great complexity that the infinitesimal rotation approximation would produce
The results of the previous sections as well as of other papers (H1 and H2) of this series can be summarized as follows: 

1. We offer a classification of extended sources of seismic radiation. The decomposition of finite sources into elementary point-sources is described. We count up to 10 different elementary sources for seismic-moment tensors which involve rotational motion. This corresponds to the low-frequency approximation of the radiation, up to \( \omega^2 \). For deviatoric extended sources the relevant number of point-sources is nine.

2. For finite earthquake sources several models of focal mechanism have been considered. We identify asperities/barriers with non-zero values of the third-rank seismic-moment tensor or with 3-D rotations of elementary dislocations (disclinations) comprising the focal zone of an earthquake. For small rotations, seven degrees of freedom are needed to describe the distribution of asperities. Three classes of geometrical barriers, corresponding to rotations around three nodal axes, are identified for this case. The most complicated model which involves 'small' barriers requires up to 20 free parameters for its characterization.

3. Statistical distributions of values of components of the third-rank seismic-moment tensor are estimated for a kinematic model of an earthquake fault. The distributions have scale-invariant features. Less than 1 per cent of the earthquake focal zones are complex enough to excite the non-standard waves with a normalized amplitude exceeding 1 per cent near, or at the 'corner' frequency.

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The results can also be used, as was mentioned in H2, for discrimination of earthquakes and nuclear explosions. Earthquake source models formulated here can be used to describe low-frequency radiation caused by complex earthquakes; these earthquakes are probably the most important obstacle in identification of the seismic events. The estimates of complexity of focal zones and assessment of frequency of occurrence for complex earthquakes discussed above provide us with a framework for such discrimination. Since estimates of multipole strengths are, in principle, orthogonal (see also Section 3.2), the discrimination criteria can be made relatively simple. However, only one source model listed in Table 4, a 'complex general' model, can be used for description of an explosion source. Clearly a catalogue of non-deviatoric models of extended sources which can be used for explosions needs to be expanded significantly. Dynamic excitation functions for half-space should also be obtained, so that an inversion of multipole sources situated near the surface could be undertaken.
REFERENCES


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