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FRACTAL PHENOMENA IN DISORDERED SYSTEMS

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INTRODUCTION

Fractal phenomena in disordered systems are now so general that a review of the past year's developments covers a broad category of topics. These include the formation of particulate aggregates, with implications for electrochemical deposition, interfaces, electrodes, contacts and membranes; dynamics of silica aerogels, and the first direct observation of quantized lattice vibrations of a fractal network (fractons); mechanical simulations of many-body model systems, and the study of nonequilibrium processes like aggregation and fracturing; multifractals, measures with the special property of self-similarity, and their use for the study of random multiplicative processes; earthquakes and fractals and the stochastic behavior of the stress prior to an earthquake; and a newly discovered phenomenon of immense importance to all of us, the fractal mechanisms of cardiac stability.

This compendium of short articles should provide the reader with a good sense of the remarkably diverse and important applications of the notion of fractal geometry. For those who wish more detailed information in a specific area, the references at the end of each article can be consulted.

The articles are grouped according to rather broad areas of interest, with the last two (on earthquakes and cardiac rhythms) focusing on newly developing fields of major personal importance. Thus, articles two through six concern diffusion limited aggregation from a variety of perspectives; seven presents very new X-ray, neutron, and optical scattering evidence.

the cluster size distribution results, while if there is no cutoff, a power-law distribution is obtained.

**Conclusions**

A variety of statistical mechanical problems are governed by random multiplicative processes. The theory of such processes is much less developed, and also less well-appreciated than the theory of random additive processes, such as random walks. Multiplicativity appears to be an essential ingredient in leading to multifractal scaling properties. The study of correlated multiplicative processes and their corresponding scaling properties should prove an interesting new area for further investigations.

**Literature Cited**


**EARTHQUAKES AND FRACTALS**

[Y. Y. Kagan, University of California, Los Angeles]

Several catalogs of tectonic earthquakes have been analyzed statistically to study interrelations between earthquakes for further theoretical understanding of the earthquake process. As first approximation, an earthquake may be represented by a sudden shear failure—appearance of a large dislocation loop (1). The catalogs characterize the set of earthquakes by origin time, hypocenter position, and by the second-rank seismic moment tensor for each event. Earthquake sequences have been treated as a tensor-valued multidimensional stochastic point process. Statistical properties of an earthquake occurrence exhibit scale-invariant features: the Pareto distribution of the earthquake size (magnitude or scalar seismic moment), power-law decay of the rate of the aftershock and foreshock occurrence (Omori's law), self-similarity of the earthquake occurrence rate in different magnitude ranges (2, and references therein). The two-, three-, and four-point moment functions of the spatial distribution of shallow earthquake
hypocenters have also been studied. The results indicate that earthquake hypocenters are concentrated on a set with the fractal dimension between 2.0 and 2.25 (3, 4). The results appear to rule out the conventional model of earthquake hypocenters occurring on a single isolated plane or on several planes, and require, instead, that an earthquake fault zone be nonplanar and fractal.

The observed self-similar patterns of an earthquake occurrence are shown to be derived from simple assumptions of the stochastic behavior of the stress prior to an earthquake. In time domain, Omori’s law of foreshock/aftershock occurrence and, in general, the time clustering of earthquake events have been shown to be a consequence of branching of earthquake fractures and Brownian motion-like behavior of random stresses (5). This means that the fractal dimension of temporal occurrence of earthquake sequences is equal to 0.5. Elaborating on Zolotarev’s result (6, section 1.1), we derive spatial patterns of earthquake fault branching from the assumption of the presence of random point defects in rocks and other materials (7). These defects rule out the planar propagation of a fault rupture. The deviations of the fault surface from planarity are characterized by a rotational Cauchy distribution that, as we have found previously, explains the nonplanar geometry of fault systems for natural earthquakes. We have demonstrated that as a result of the 3-D rotations of elementary dislocations comprising the earthquake source zone, the resulting complex extended source should contain rotational dislocations (disclinations) that we identify with asperities/barriers controlling the initiation, propagation, and stopping of earthquake fractures (8).

On the basis of stochastic modeling, a model of an earthquake occurrence has been developed to simulate the earthquake process by the Monte-Carlo method (2, 8, and references therein). In this model an earthquake fault pattern is assumed to consist of a system of small elementary plane dislocations. Following the initial dislocation, subsequent ruptures occur according to a stochastic critical branching process. The position of each secondary dislocation is randomly shifted, along the fault-plane of its predecessor, from the location of the main shock. In addition, the orientations of the fault-plane and slip vector of the secondary dislocations are rotated according to the three-dimensional Cauchy distribution. The simulated fault pattern is both visually and statistically similar to real earthquake faults. We use the above results to identify a foreshock sequence in progress in order to develop a method of online earthquake prediction (2). As a predictor, the procedure reduces the average uncertainty in the rate of occurrence for a future strong earthquake by a factor of more than 1000 when compared with the completely random (Poisson) rate of occurrence.
DYNAMICS OF SUDDEN DEATH

[A. L. Goldberger and D. R. Rigney, Harvard Medical School]

Each year, tens of thousands of Americans die suddenly from a disturbance of their heart rhythm (cardiac arrhythmia). New understanding of the mechanisms of both normal cardiac function and sudden cardiac death comes from the application of nonlinear dynamics and fractals to physiology and medicine (1–5). These nonlinear concepts challenge certain deeply ingrained preconceptions regarding the dynamics of healthy function and disease.

Under healthy conditions, the heartbeat (pulse rate) is controlled by the firing of pacemaker cells in the sinus node located in the right atrium. Physicians and physicists often assume that the healthy heartbeat is highly regular, as implied by the clinical term regular sinus rhythm. Closer inspection of heartbeat time series data from healthy subjects at rest, however, reveals an unanticipated finding, namely, cardiac chaos. The heartbeat in normal individuals even at rest fluctuates in an erratic fashion. Further, the spectrum of this process is broadband, with a 1/f-like distribution (Figure 10). Based on these findings we have proposed that heartbeat is under the control of a nonlinear, fractal feedback system that generates self-similar fluctuations across multiple orders of temporal magnitude. The details of this fractal mechanism are currently being investigated. An essential feature appears to be the nonlinear interactions between the sympathetic and parasympathetic branches of the autonomic (involuntary) nervous system that regulates the firing rate of sinus node pacemaker cells.

This counterintuitive chaos theory of healthy variability is supported by two additional lines of evidence. The fractal nature of healthy function is