1. Introduction

[2] The state of stress is considered a most important parameter for controlling the occurrence of earthquakes. Recently, there has been interest in statistical analysis of stress patterns and their relation to earthquakes [Saucier et al., 1992; Reasenberg and Simpson, 1992; Kagan, 1994a; Gross and Kisslinger, 1994; King et al., 1994; Stein et al., 1994; Harris et al., 1995; Harris and Simpson, 1996; Jaumé and Sykes, 1996; Stein et al., 1997; Deng and Sykes, 1997a, 1997b; Stein, 1999; Ziv and Rubin, 2000; Parsons et al., 2000; Parsons, 2002; Huc and Main, 2003; Lin and Stein, 2004; Steacy et al., 2004; Hardebeck, 2004; Pollitz et al., 2004; Helmstetter et al., 2005]; see also a review by Harris [1998]. Many of the stress investigations are concentrated in California. This is due to the accessibility of California earthquake faults to direct geologic study, including the measurement of fault slip rates and paleoseismic investigations of large past earthquakes. Geodetic measurements of surface strain have been available since the middle of the 19th century. A recently installed dense network of GPS stations allows for detailed mapping of tectonic deformation in California [Shen et al., 1996; Jackson et al., 1997]. Moreover, the first local seismographic network installed in southern California continues operation [Hileman et al., 1973]. Thus California offers a unique opportunity to study the relationship between stress and earthquakes using geologic, geodetic, and seismological data.

[3] Most publications above investigate stress triggering for sequences of moderate and small earthquakes. However, tectonic loading due to lithospheric plate motion and its release by earthquakes have not been fully studied. The short-term influence of stress perturbations caused by earthquakes is obvious: aftershocks are usually explained by such influence. The above references strongly delineate intermediate-term stress triggering of earthquakes by recent events. However, on timescales of decades and possibly even years, tectonic stress loading will play a major role in earthquake occurrence.

[4] Despite such long and intense study, there are still some difficulties in understanding the interaction of stresses and earthquakes in California. Its stress environment was strongly influenced by great earthquakes (with magnitude $M \geq 7.75$). For two such quakes, the 1857 Fort Tejon and 1906 San Francisco events, we have sufficient data on the slip pattern and earthquake occurrence before and after. One
can conjecture that previous great quakes shaped the California stress environment to a large degree, but unfortunately we have only incomplete information on these prehistoric events from paleoseismic investigations [see, e.g., Weldon et al. [2002, and references therein]. In effect, this means that we are able to study only one or two events that modified regional stress, since in general each earthquake exhibits significant random fluctuations. One cannot be sure that patterns shown by these great earthquake sequences would be applicable to other global events, or to future quakes in California.

Another problem we have to address is the influence of small earthquakes on stress. Helmstetter et al. [2005] argue that since earthquakes are spatially clustered, the combined influence of small earthquakes is similar to that of large events. The degree of spatial clustering can be measured by the correlation dimension [Kagan and Knopoff, 1980] for earthquake hypocenters. If the correlation dimension is equal to 2, the influence of small earthquakes within a unit magnitude band approximately equals that of large earthquakes in a similar band [Helmstetter et al., 2005]. Similarly, Hanks [1992] argues that for planar faults small earthquakes are just as important to redistribute tectonic forces as larger ones. However, if the accuracy of earthquake hypocenters is low, the correlation dimension estimate increases to about 3.0 for distances comparable to location error [Kagan, 1994b]. If this distance is larger than the size of the earthquake focal region, large earthquakes would appear to dominate the stress pattern. Therefore, even if small events are important in estimating the stress-earthquake relation, available data do not permit a meaningful investigation.

2. Data

Figure 1 displays focal mechanisms for the earthquakes in southern California between 1800 and 2003. As the initial data set we use historical and instrumental earthquake catalogs by Ellsworth [1990] and Toppozada et al. [2000]. Our catalog covers the years 1800–2003, and
an area defined by 32.0–37.0°N and 114.0–122.0°W. We added (1) recent earthquakes from the Harvard catalog [Ekström et al., 2003], (2) the focal mechanism solutions and spatially distributed seismic moment from other available publications [Bäth and Richter, 1958; Fehler and Johnson, 1989; Heaton, 1982; Helmberger et al., 1992; Hileman et al., 1973; Hill et al., 1990; Hutton and Jones, 1993; Stein and Thatcher, 1981; Stein and Ekström, 1992; Toppozada et al., 1986; Wesnosky, 1986; Toppozada et al., 2000; Pasyanos et al., 1996], and (3) distributed moment tensors inferred from the fault trace information [Jennings, 1985, 1992] and slip distribution [Bateman, 1961; Sieh, 1978] for the largest earthquakes in the 19th century.

[7] We included all known earthquakes with $M \geq 5.0$ and represent any earthquake with $M \geq 6.5$ as an ensemble of rectangular dislocations. We also added to the Ellsworth and Toppozada et al. catalog information on the rupture pattern of the southern part of the 1906 earthquake [from Thatcher et al., 1997] and the 1812 earthquake [from Deng and Sykes, 1997a] to create a catalog starting from 1800. We include earthquakes $6.0 \geq M \geq 5.0$ in the last 65 years from Deng and Sykes [1997b] as well as from other sources [Stein and Hanks, 1998]. For many of these ($6.0 \geq M \geq 5.0$) earthquakes, the focal mechanism is unknown; we estimated their mechanisms using a weighted average from nearby earthquakes with known focal mechanisms. In almost all cases, these quakes are large events ($M \geq 6.5$) for which either their fault traces are known, or the fault plane is delineated by aftershock pattern and surface deformation measurements. Hence, by comparing focal mechanism solutions to rupture patterns of large earthquakes, for practically all moderate and small earthquakes we could guess the fault plane and resolve the fault plane ambiguity. This catalog is called the Ellsworth/Toppozada catalog below. The catalog is available on line; the URL is http://moho.ess.ucla.edu/~kagan/cal_fps2d.dat.

[8] Stein and Hanks [1998] argue that because of the low population level in southern California during most of the second half of the 19th century the catalog may be incomplete for earthquakes $M \leq 6.5$. However, numerical experiments show that adding and subtracting even a few in the range $6.0 \leq M \leq 6.5$ during the time period 1850–1900 would not affect the results very much.

[9] In Figure 1 the extended sources are thinned out for clarity; otherwise the picture would be overloaded. In the full data set (the Ellsworth/Toppozada catalog) there are 381 “sources,” including point sources for smaller earthquakes and subdivision of $M \geq 6.5$ earthquakes. For the “thinned” catalog only 167 double couples are displayed in the diagram. Figure 1 shows that earthquakes are not concentrated on a few faults and the mechanisms of neighboring events may have very different orientations. Even in a neighborhood of major faults, some focal mechanisms significantly disagree with fault surface traces. This mismatch confirms the idea that major faults do not fully represent the deformation pattern, even in a region with relatively simple and well-studied tectonics. The thinned catalog also is available on line; the URL is http://moho.ess.ucla.edu/~kagan/cal_fps3a.dat.

[10] Several other catalogs of fault plane solutions are used in this study: the list of southern California earthquakes 1968–1993 by Harris et al. [1995], L. M. Jones’ (private communication, 1993) catalog of earthquakes 1986–1993, and the list of 1990–1995 moment tensor inversions of Terrascope records [Thio and Kanamori, 1995, 1996; Zhu and Helmberger, 1996], the University of California, Berkeley, moment tensor (UCB-MT1) catalog [see Pasyanos et al., 1996; Tajima et al., 2002]. Several solutions for one earthquake are sometimes presented in the last catalog. We use the solutions based on surface waves (called UCB-MT1), and another set of solutions, derived from a three-component inversion (UCB-MT2). For the California Institute of Technology focal mechanism (CITFM) 1975–1999 catalog, E. Hauksson (personal communication, 2001) supplied a revised data set for earthquakes with magnitude $M \geq 4.9$ [Hardebeck and Hauksson, 2001; Hauksson, 2000]. Hardebeck’s 1981–2000 catalog (J. L. Hardebeck, private communication, 2003) [see also Hardebeck and Shearer, 2002], based on a new method for determining first-motion focal mechanisms, was also used. Kagan [2002] describes the accuracy of focal mechanism evaluation for most of the above mentioned catalogs.

3. Stress Calculations
3.1. Tectonic Stress Calculations

[11] We derived a theoretical estimate of the stress rate tensor at each point by (1) estimating the vector displacement rate from a “back-slip” deformation model, (2) estimating the strain rate tensor from a spatial derivative of the displacement rate, and (3) estimating the stress rate by applying Hooke’s law to the strain rate. The method is described by Ge [1997].

[12] The deformation model was constructed to explain the observed fault slip rates and geodetically observed velocities in southern California. In the model, the Earth’s crust is made up of blocks bounded by faults. Several other California block models have been proposed [e.g., Matsu'ura et al., 1986; Cheng et al., 1987; Bird and Kong, 1994; Ward, 1996]. Other applications of the back-slip model are described by Dragert et al. [1994], Hashimoto and Jackson [1993], Henry et al. [2001], and Mazzotti et al. [2000].

[13] We begin by subdividing the study area into a few dozen blocks, bounded by the major faults. We treat the tectonic motion of any point on the Earth’s surface as the sum of the steady state rigid motion of its underlying block and elastic deformation of that block due to frictional forces on the block-bounding faults. Rigid block motion is computed by the plate theory of McKenzie and Parker [1967], in which the motion of each block is represented by a rate of rotation about an Euler pole. The rigid motion of each block is fully specified by three parameters, which could be the latitude and longitude of the Euler pole plus the rotation rate around it or, equivalently, three orthogonal components of the rotation vector.

[14] If the blocks were indeed rigid, then the theory would imply a velocity discontinuity (slip) at each block boundary. However, of course, the blocks are not rigid, and most faults only slip in major earthquakes. There are some exceptional “creeping” faults that do slip continuously, although the slip rate is not necessarily equal to the rate implied by the rigid block model. To match the observed zero slip rates on faults except for the creeping ones, we
assume at each point on a fault a dislocation whose slip rate equals the difference between the creep rate and the velocity discontinuity implied by the rigid block model. In most cases the creep rate is zero, and the dislocation rate is the negative of the slip rate implied by the rigid block model. The dislocation then causes elastic deformation in the block which we model using the theory of Okada [1992]. Assuming that deformation is indeed elastic, we can then compute the stress rate using Hooke's law. The dislocation motion can be viewed as a correction to rigid body motion and is often referred to as the “back slip” because it is in the opposite sense to that implied by rigid block motion. In practice we treat the back slip as uniform slip on rectangular fault patches. The discontinuity in rigid block motion will vary with distance along a fault, so uniform slip on a patch cannot cancel the rigid block motion at every point on the patch. However, this discrepancy can be made arbitrarily small by using small enough patches. The relative motion implied by the rigid block model will also include a normal component, because the faults cannot be everywhere parallel to the relative block motion. We treat the normal component in the same way as the tangential components: we apply a normal dislocation to cancel the rigid block motion. This allows for the equivalent of creep where there is evidence of overthrusting or basin opening on particular faults.

[15] We assume that over geologic time earthquakes release all accumulated back slip. Thus the long-term slip rates on faults provide direct estimates of relative rigid block motion. Another consequence is that the back slip can also be described as a “slip deficit” to be made up over the long term by earthquake slips.

[16] With the assumptions above, we inverted a combination of the fault slip rates and geodetic velocities estimated by the Southern California Earthquake Center (SCEC) to estimate block translation and rotation rates, from which we estimated strain rates and stress rates throughout the crust. The fault slip rates are reported in the SCEC Phase II model [Jackson et al., 1995]. The geodetic data include trilateration, Global Positioning System, and very long baseline interferometry data, nearly identical to the data used to construct the SCEC version 1.0 of the SCEC crustal deformation model (available at http://www.data.scec.org/group_e/release.v2).

[17] The back-slip model does have several unrealistic features, especially when the block boundaries are assumed to be “tiled” with rectangular dislocation patches. The model implies displacement discontinuities and thus stress singularities at patch boundaries. Rectangular patches cannot be joined at their vertical edges unless the patches are strictly vertical. These deficiencies may be minimized by taking arbitrarily small patch size and by using the numerical calculations only in the far field (away from the patch edges). Cohen [1999] examines the features of the model in some detail.

3.2. Seismic Stress Calculations

[18] As we indicate in section 2, our catalog consists of two sets of earthquakes: small events ($M < 6.5$) that are represented as point sources and large earthquakes ($M \geq 6.5$). The latter events’ rupture surface is divided into several (sometimes several dozen) rectangular dislocations. Depending on available information, these patches are distributed along the length of a fault and depth of a seismogenic zone.

[19] The seismic moment $M_0$ can be expressed through shear elastic modulus $\mu$, average slip $u$, and width and length of the rupture, $W$ and $L$, respectively,

$$M_0 = \mu uWL.$$  

For small earthquakes we commonly used a point representation of the rupture. In other cases, assuming the ratio of rupture length to average slip $L/u = 10^4$ (we experimented with other ratio values as well), we calculate the rupture area $L \times W$ of small events using their moment value. Thereafter, by assuming their rupture depth interval, we represent them as a single rectangular dislocation. In most cases, if the depth interval is not independently known, we use 10 km for both small and large earthquakes, centered at the hypocenter. If the hypocenter depth is not known, as is common for old historical and instrumental earthquakes, we assumed it to be 10 km. For small earthquakes with a hypocenter shallower than 5 km, the calculated rupture depth interval sometimes intersects the Earth’s surface; in such a case we shifted the interval down. Thus, to evaluate the stress tensor due to earthquakes, we used both point representation of a seismic source as well as a rectangular model of a fault patch [Okada, 1992]. The shear elastic modulus value is taken as 30 GPa [Scholz, 2002, p. 207]. In our stress moment correlations we compute stress at the location of the quake hypocenter for small earthquakes but at the center of the dislocation patch for large events.

4. Stress Patterns and Focal Mechanisms of Earthquakes

[20] Figure 2 shows the N-S component ($\tau_{11}$) of the cumulative stress tensor for southern California due to earthquakes from 1850 to the present (1 January 2004). At large distances, the stress distribution is dominated by the influence of several large earthquakes: the 1857 Fort Tejon, 1872 Owens valley, 1952 Kern County, and 1992 Landers events. The seismic stress pattern forms a complex mosaic due to the interacting seismic stress fields of many earthquakes [cf. Stein et al., 1992]. The complicated character of stress once again underscores the need for statistical data analysis.

[21] Figure 3 displays the cumulative stress change again for $\tau_{11}$ component due to tectonic deformation since 1850. The tectonic model unifies geodetic and geologic data [Shen et al., 1996] (see section 3).  

[22] To compare tectonic and earthquake-induced stresses, Figure 4 shows mean square shear stress $\bar{\tau}$ or average shear stress for the sum of seismic and tectonic stresses. The mean square shear stress is calculated as

$$\bar{\tau} = \sqrt{2J_2/5},$$

where $J_2$ is the second invariant of the deviatoric stress tensor [Jaeger and Cook, 1979, pp. 24, 33; Kagan, 1994a, equation 6]. The influence of tectonic stress is prevalent in the combined plot. Comparing Figures 1 and 4, we see that
earthquakes occur where total (seismic plus tectonic) stress is high.

We compute normal and shear stress on nodal planes for each earthquake in the catalog. We compare the focal mechanisms of quakes with the resolved shear and resolved compression before the events at their hypocenters. The fault plane determination for large ($M \geq 6.5$) earthquakes is obvious; we have a rupture model for each event. To determine which of two nodal planes is the fault plane for small earthquakes, we compared their planes with the rupture direction of neighboring large earthquakes. The nodal plane more similar to large event ruptures was selected as the fault plane.

How do we correlate earthquake focal mechanisms with stress? There are significant theoretical and practical difficulties to such a comparison. First, both the focal mechanism and the stress are symmetric second-rank tensors. Focal mechanism tensors for double-couple sources have certain restrictions: their trace and determinant (i.e., the first and third tensor invariants) are both zero. The stress tensor lacks such restrictions. Instead of the full tensorial consideration, we may calculate the normal stress $s$ and shear stress $t$ on the assumed or known earthquake fault plane. As an alternative, the tensor invariants [Kagan, 1994a, 1994b] can be studied; invariants do not depend on the coordinate system in which the tensor components are represented. For example, the stress display in Figure 4 does not depend on the coordinate system used.

Second, the component values of both stress and seismic moment tensors are heavy-tailed [Samorodnitsky and Taqqu, 1994; Kagan, 1994a, 1994b; Uchaikin and Zolotarev, 1999; Lavallée and Archuleta, 2003; Zaliapin et al., 2005]. Random variables are called “heavy-tailed” if their probability density function $\phi(x)$ has a power law dependence for large values of the $x$ variable

$$\phi(x) \propto x^{-1-\alpha},$$

with $\alpha < 2$. For such a statistical distribution, the variance does not exist, and the mean is infinite for $\alpha < 1$. The theoretical and practical problems in handling the heavy-tailed (or stable) distributions is a rapidly developing, but still incomplete discipline in mathematical statistics [Samorodnitsky and Taqqu, 1994; Uchaikin and Zolotarev, 1999; Zaliapin et al., 2005].

The collection of scalar seismic moments ($M_0$), proportional to the tensor norm, is well approximated by the tapered Pareto distribution (the equivalent of the modified Gutenberg-Richter law) [Kagan and Jackson, 2000; Bird and Kagan, 2004]. Components of the seismic stress tensor, calculated at the location of earthquake hypocenters, follows stable distributions [Kagan, 1994a, 1994b; Lavallée and Archuleta, 2003] with the exponent value $\alpha \leq 1.0$. Unfortunately, as mentioned above, classical statistical tools like mathematical expectation (average), covariance, and correlation coefficient are not defined when $\alpha \leq 1.0$.

Instead of covariance, Samorodnitsky and Taqqu [1994, p. 103] propose for such variables the “codifference.” When $\alpha = 2.0$, the stable distribution becomes the Gaussian one; for this variable the codifference equals the

![Figure 2. Earthquake-induced stress in southern California at the Earth’s surface. The modified Ellsworth/Toppozada catalog 1850–2004 is used. Horizontal (N-S) stress component $\tau_{11}$ is shown.](image-url)
Figure 3. Cumulative tectonic stress change in southern California, since 1850, calculated using elastic blocks, delineated by faults, at the Earth’s surface [Ge, 1997]. Horizontal (N-S) stress component $\tau_{11}$ is shown.

Figure 4. Shear stress in southern California at the Earth’s surface. The modified Ellsworth/Toppozada catalog 1800–2004 is used. Mean square seismic plus tectonic shear stress is shown.
Table 1. Resolved Stress Correlation With Focal Mechanisms of Earthquakes

<table>
<thead>
<tr>
<th>Case</th>
<th>Catalogc</th>
<th>Correlation Δσij</th>
<th>Time Limit</th>
<th>Distance Limit</th>
<th>Stress Limit</th>
<th>σij</th>
<th>Ratio</th>
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<td>a</td>
<td>0.15 ± 0.07</td>
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<td>--</td>
<td>&gt;500</td>
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<td>--</td>
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7Ratio signifies that ratio of earthquakes with positive stress moment correlation to total event numbers is shown.  
8U, uniform N-S compression/E-W dilatation stress; UN, uniform N-S compression/E-W dilatation stress, no seismic stress; B, stress calculated using the block model.  
9σij is a normalized double-couple moment tensor as in [30]. Let us consider the tensor dot product of two seismic moment tensors \( S_{ij} = \sum M_{ij}S_{ij} \) displayed in their eigenvector coordinate system, for the seismic moment of double-couple source, and  

\[
M_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]  

\[
S_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{pmatrix},
\]

for the deviatoric stress tensor, where \( |A - 1| \leq |A| < 1 \) is assumed.  
10Let us consider the tensor dot product of two normalized tensors  

\[
P = \sum_{pq} \sigma_{pq}m_{pq}.
\]
2 if an earthquake is fully consistent with stress, and but −2 for an anticonsistent event.

[31] Table 1 shows some average tensor dot products (8) of stress and seismic moment $P$ for several catalogs of focal mechanisms. We performed several tens of such calculations, with a few representative results displayed in Table 1. We also show $(\sigma_P)$, the standard deviation of $P$.

[32] As a statistical test, we compare the difference between two $P$ values with their standard deviations, $\sigma_P$. For statistically independent data, the ratio

$$z = \frac{P_1 - P_2}{\sqrt{\sigma_1^2 + \sigma_2^2}},$$

(9)

is distributed for a large number of events ($n > 30$) according to a normal (Gaussian) distribution with a standard deviation of 1. This means that if $|z| \geq 1.96$, the hypothesis of $P$ equality can be rejected at a 95% statistical significance level.

[33] Clearly, the components of extended earthquake sources are statistically dependent. Even if we use a point source catalog, earthquakes are clustered in space, time, and focal mechanism space [Kagan and Jackson, 1994]. Given the positive correlation of focal mechanisms for nearby earthquakes, the significance of small differences in $z$ will be underestimated if we neglect the correlation. Thus, whereas large values of $z$ indicate that $P$ values are statistically different, small $z$ values may be due to data dependence.

[34] We investigated the influence of catalog selection, catalog time and space limits, temporal delay between earthquakes ($\Delta T$), and tectonic stress on the correlation values. Following Harris et al. [1995], we also tried to exclude from calculation pairs of earthquakes within a close distance in space ($R$) and earthquakes with a stress $\sigma$ value below a threshold 0.001 or 0.01 MPa (0.01 or 0.1 bar). We did so because spatial concentration of hypocenters can be due to location errors, and such earthquakes may induce spurious high stress increase. We remove in some cases earthquakes with small stress value, since it is unlikely that such stress would influence earthquake triggering.

[35] From Table 1 we see that the low correlation between seismic stress and seismic moment strongly depends on catalog choice and parameters of calculations. In general, selecting earthquake pairs with a temporal separation of less than 500 days increases the correlation level significantly or nearly significantly for three catalogs (cases 3–5, 10–11, 17–18), and insignificantly for three catalogs (cases 7–8, 19–20, 22–23). In the last two catalogs the correlation has an opposite sign, but the $|z|$ values (9) are less than 1.96. The influence of $R$ and $\sigma$ on the correlation is not obvious. Comparing cases 1–12 of Table 1, we see that depending on the catalog form, correlation values may even change their sign due to random fluctuations. Random fluctuations of the $P$ value on the order 0.2–0.3 are quite common for catalogs of a few hundred or a few dozen events.

[36] Cases 16–23 in Table 1 explore the influence of temporal delay between earthquakes [Harris et al., 1995]. Contrary to Harris et al. [1995], we calculate the total resolved shear seismic stress due to all prior earthquakes. For Harris et al. [1995] catalog, the results broadly confirm their conclusions: the correlation value is significantly higher for time separation less than 1.5 years compared to longer time delays between earthquakes. However, if we use other catalogs, the results are not that clear cut: the tendency in the Ellsworth/Toppozada catalog is opposite to that of Harris et al. [1995], whereas there is practically no dependency of $P$ on $\Delta T$ for L. M. Jones’ (personal communication, 1993) catalog.

[37] Cases 13–15 and 25 in Table 1 demonstrate the influence of tectonic (aseismic) stress on the correlation between the total stress prior to an earthquake and the seismic moment tensor of the event. The value of $P$ is significantly higher when we include tectonic stress. It turns out that the result does not depend strongly on whether we use a very simple model of the tectonic stress increments (uniform N-S compression/E-W dilatation stress) or a more sophisticated one based on a block model of southern California displacement [Ge, 1997]. In the former case, adding the seismic to the tectonic stress degrades the correlation between the total stress and focal mechanisms of earthquakes (event 14 of Table 1).

[38] Cases 29–33 in Table 1 show results for a few recent focal mechanism or seismic moment tensor catalogs. Only for an extensive focal mechanism catalog (J. L. Hardebeck, private communication, 2003) is there any positive correlation/triggering between seismic stress and earthquakes. This dependence needs to be studied in more detail.

[39] We analyzed the influence of the Fort Tejon 1857 earthquake on subsequent activity [Harris and Simpson, 1996]. Our calculations differ from those by Harris and Simpson [1996]: we use a larger spatial window (32.0–37.0°N, 114.0–122.0°W) and another (thinned) version of the Ellsworth/Toppozada catalog. The results displayed in cases 26–28 suggest that there is no obvious “stress shadow” in early years after the 1857 earthquake. The correlation values do not exhibit a clear pattern. Felzer et al. [2003] obtained a similar result.

[40] We repeated the computations of Harris and Simpson [1996] using different versions of the Ellsworth/Toppozada catalog. We also added tectonic stress to the static stress from the 1857 event, making the spatial window similar to that of Harris and Simpson [1996] and restricting stress to values above 0.001 or 0.01 MPa (0.01 or 0.1 bar), etc. Again no obvious pattern emerges. Stein and Hanks [1998] suggest that the stress shadow pattern may be an artifact due to the catalog incompleteness during the second half of the 19th century.

5. Statistical Distributions of Stress

[41] Figures 5–8 display examples of the cumulative statistical distributions of stress resolved on earthquake nodal planes prior to events (prestress). To make plots more graphically clear, in the cumulative plots we assume that the first entry corresponds to all earthquakes having resolved seismic normal stress $\sigma$ less than or equal to −1 MPa. Similarly, the last entry counts the normalized number of events with $\sigma \geq 1$ MPa.

[42] Distribution of the normal stress (Figure 5a) implies that the friction coefficient is small. According to the standard view [e.g., Scholz, 2002], we would expect the distribution of normal stress $\sigma$ to be highly asymmetric with
more earthquakes occurring when seismic stress is dilatational. Contrary to such expectations, the cumulative curves in Figure 5a exhibit no preference for a dilatational prestress (for example, 5% of earthquakes occur when $s/C_21 \geq 1$ MPa, about the same number as for $s/C_20/C_0 \geq 1$ MPa). The curves for tectonic stress display a similar behavior; they are roughly symmetrical with regard to the point (0, 0.5).

The asymmetry of dilatational versus compressional stress would be easier to see on distribution histograms. Unfortunately, because of a relatively small number of test earthquakes, such histograms exhibit large random fluctuations, making conclusions uncertain. Hence, to better demonstrate subtle asymmetries of stress distributions, we constructed Figure 5b and several similar diagrams which show distribution difference ($D$) between negative and positive values of the normal stress. We calculate $D = 1.0 - F(-a) - F(a)$, where $F(a)$ is the value of a cumulative function corresponding to normal stress level $a$. If a stress distribution is symmetric with regard to the stress zero value, the above expression is zero. Let us first consider the seismic stress. The diagram shows some asymmetry: for small values of $\sigma$ (0–0.3 MPa), more than 10.0% of earthquakes occur in regions of compression than in dilatational zones, i.e., contrary to the Coulomb law. A commonly accepted model suggests that the Coulomb failure stress change $\sigma_f$ (Scholz, 2002) controls earthquake occurrence:

$$\sigma_f = \tau + \mu_f \sigma_n,$$

where $\tau$ is the seismic shear stress on a fault plane, $\mu_f$ is a static (positive) coefficient of friction, and $\sigma_n$ is a normal stress change (positive $\sigma_n$ corresponds to relative extension).

Failure stress change $\sigma_f$ controls earthquake occurrence:

$\sigma_f = \tau + \mu_f \sigma_n,$

where $\tau$ is the seismic shear stress on a fault plane, $\mu_f$ is a static (positive) coefficient of friction, and $\sigma_n$ is a normal stress change (positive $\sigma_n$ corresponds to relative extension).

Figure 5. Statistical distributions of normal stress $\sigma_n$ resolved on earthquake nodal planes prior to earthquakes in the Ellsworth/Toppozada catalog (1850–2004). In all plots, the solid line is for the seismic stress due to earthquakes, the dash-dotted line is for the seismic plus uniform tectonic stress, and the dashed line is for the seismic plus tectonic stress, according to block model of displacement. (a) Cumulative distribution for the stress normal to the fault plane. Dotted line is for the Cauchy distribution. The negative stress values correspond to compression. (b) Difference between cumulative distributions for negative and positive values of the normal stress (see Figure 5a). Absolute value of the stress is abscissa of plot. Values above zero correspond to prevalence of dilatation over compression for resolved stress at centroids.

Figure 6. Statistical distributions of shear stress $\tau$ resolved on earthquake nodal planes prior to earthquakes in the Ellsworth/Toppozada catalog (1850–2004). In all plots, the solid line is for the seismic stress due to earthquakes, the dash-dotted line is for the seismic plus uniform tectonic stress, and the dashed line is for the seismic plus tectonic stress, according to block model of displacement. Plots are similar to Figures 5a and 5b, but the resolved shear stress is plotted. (a) Resolved shear stress, the stress negative values correspond to focal mechanisms inconsistent with the stress. (b) Resolved shear stress difference, the values above zero correspond to resolved stress be consistent with a focal mechanisms at centroids.

Figure 7. Difference between cumulative distributions for negative and positive values of the normal stress (compare to Figure 5b) for the Ellsworth/Toppozada catalog (1932–2004). All lines and diagrams are similar to Figure 5b.
of the normal stress (as in Figure 5b) for different time windows. Obviously the general behavior is similar to Figure 5b. For seismic stress and for block models of tectonic stress, the values of the distribution do not display any consistent pattern: the numbers of earthquakes triggered in the dilatational volumes approximately equals the numbers in the compressional parts. Only the uniform model of tectonic stress exhibits a certain preference for dilatational triggering. With available data it is difficult to understand the causes of such behavior.

Similarly, in Figure 8 we show the influence of shear stress triggering for the same selection of catalogs as in Figure 7. One can draw the same conclusion from these displays as that proposed above (Figure 6b): both tectonic stress curves support the idea that earthquakes are triggered by aseismic stress accumulation. Experimenting with various catalogs and modified assumptions shows that in most cases cumulative distributions may change by a few percent, while the major features of the distribution remain robust. The results of analyzing the statistical distributions broadly agree with the results from section 4.

6. Discussion

Stress is believed to be the most important variable controlling triggering, earthquake occurrence, and interactions. Many interesting results have been obtained thus far (see discussion in section 1 and the review by Harris [1998]), but interpreting them in the framework of stress accumulation and release using recurrence models encounters difficulties. If a large earthquake occurs when the stress exceeds the strength of rocks, why do small earthquakes occur over the seismogenic zone all the time? If the stress value is close to the critical level over a large area, should strong earthquakes occur more frequently than usual, leading to a smaller $b$ value in the region? This feature has not been observed unambiguously. Or does the increased stress level simply trigger more earthquakes without regard to size? The magnitude-frequency relation for aftershocks does not seem to vary from that for all earthquakes. Does the local stress at the hypocenter control an earthquake or does regional stress over the rupture zone? What stops the progression of earthquake rupture?

If there is a stress shadow after a large earthquake in the focal zone and nearby, how can one explain aftershocks? There is no clear spatial, temporal, or magnitude boundary between the aftershocks and other earthquakes. If the increased stress triggers earthquakes and a large earthquake releases stress, why are there more aftershocks than foreshocks? Many models of stress triggering assume that the Coulomb fracture criterion is also valid for the Earth’s interior, as has been established in laboratory testing of rock specimens. However, numerous attempts to evaluate the friction coefficient in situ have been inconclusive, often suggesting that the coefficient is close to zero [Bird and Kong, 1994; Kagan, 1994a; Harris, 1998].

In the investigations reported here, we tried to answer some of the above questions. Statistical analysis suggests a poor reproducibility of results: insignificant changes in input data or data-handling assumptions substantially modify conclusions. Although in many cases we have been able to approximately reproduce the results of other researchers by...
using the same data, applying similar procedures to another set of data usually leads to different results. Both the data studied and the assumptions are arbitrary, necessitating a subjective approach to their interpretation.

[51] Catalog incompleteness and uncertainties in earthquake source parameters limit our ability to understand the relation between seismic stress and earthquake occurrence. Helmstetter et al. [2005] and others have shown that earthquakes below the completeness threshold of available catalogs might affect the stress as much as those in any available catalog. Large earthquakes before the start of the catalog may also overwhelm the stress from cataloged events. Thus incompleteness seriously limits our understanding of triggering. Our results show that source uncertainty is also a serious problem. Even if uncataloged events had no effect on the stress, the uncertainties in location, size, focal mechanism, and slip distribution of cataloged events would allow for very different interpretations of the triggering by seismic Coulomb stress. Some recently available catalogs provide much improved precision, at least for smaller events (see Helmstetter et al. [2005] for a discussion). Perhaps in the future we can assess triggering with much greater confidence.

[52] These results suggest that the evidence for earthquake triggering by seismic stress is weak. There are, among others, two reasons for this: (1) low spatial resolution of available data, and (2) a possibility that the stress acting in a focal zone may result from many large earthquakes which occurred some time ago, and we have little, if any, information about them. This apparently weak stress influence in turn makes it difficult to study using only a few case studies. A weak signal can be easily misinterpreted or biased by using various modeling assumptions and different subsets of data. Perhaps the only way to obtain reliable results is to analyze large data sets statistically or to use earthquake data sets with significantly higher location and focal mechanism accuracy.

[53] In the preceding paragraph we mention that the evidence for earthquake triggering by seismic stress is weak. This does not by necessarily mean that the seismic stress is inconsequential in earthquake triggering. We can only compare estimated moment tensors with estimated stress tensors, and both estimates are limited by measurement errors and the adequacy of available data.

[54] For this work we adopted a measure of correlation between a preexisting stress tensor and an earthquake moment tensor. This criterion differs from that used by many other investigators, who often use a simple binary score based on the sign of the stress moment correlation. Is it possible that we obtain different results from others because of our different criterion? Examination of Table 1 shows that that is not likely. The ratio (number/total) is essentially the traditional criterion, and it agrees well with our correlation. Those tests that result in a high correlation also result in a high ratio of favored to unfavored earthquake mechanisms, and the same for low correlations. Our correlation method is more robust because it is based on the size, not just the sign, of the correlation between stress and moment.

7. Conclusions

[55] We analyzed the relationship between earthquake focal mechanisms and preexisting stress as an index of earthquake triggering. We included in preexisting stress tectonic (aseismic) stresses and seismic stresses (from past earthquakes), both separately and in combination. Results, given in Table 1, can be summarized as follows:

[56] 1. The most robust relationship is that earthquake focal mechanisms are consistent with the tectonic stress, whether that is estimated from a simple uniform stress model or a much more detailed model based on faults and blocks.

[57] 2. Earthquake focal mechanisms are moderately well correlated with the seismic stress from earthquakes within the previous 500 days. However, this correlation is sensitive to arbitrary choices in data selection like the start time, the maximum distance considered and the maximum stress considered. Also, adding seismic stresses does not significantly improve the correlation, compared with a model based on tectonic stresses alone.

[58] 3. There is no significant correlation between earthquake focal mechanisms and the stress left by earthquakes more than 500 days before.

[59] 4. The normal component of stress has little influence on earthquake occurrence. If Coulomb stress is important in earthquake triggering, the effective coefficient of friction must be indistinguishable from zero.

[60] 5. Our results are generally consistent with other published results when we make the same choices in data selection; when we make other reasonable choices, we often get rather different results for the effect of seismic stresses. The sensitivity to arbitrary choices is due to natural variability, or to insufficient information about the stress field and focal mechanisms, rather than differences in calculation methods.

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References


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